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# DERIVATIVES PRICING USING QUANTLIB: AN INTRODUCTION

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#### Abstract

Given the complexity of over-the-counter derivatives and structured products, almost all of derivatives pricing today is based on numerical methods. While large financial institutions typically have their own team of developers who maintain stateof-the-art financial libraries, till a few years ago none of that sophistication was available for use in teaching and research. For the last decade, there is now a reliable C++ open-source library available called QuantLib. This note introduces QuantLib for pricing derivatives and documents our experience using QuantLib in our course on Computational Finance at the Indian Institute of Management Ahmedabad. The fact that it is also available (and extendable) in Python has allowed us to harness the power of C++ with the ease of iPython notebooks in the classroom as well as for student's projects.

 $Keywords\colon$  Derivatives pricing, Financial engineering, Open-source computing, Python, QuantLib

# 1 Introduction

Financial engineering and algorithmic trading are perhaps two of the most computationally intensive parts of all of finance. A job in either of these areas requires not only a reasonable expertise in finance, mathematics and statistics, but also, and perhaps more importantly, a sophistication in computing.

While algorithmic trading is about finding opportunities in markets that give a temporary statistical edge - so the focus is on speed and data-mining, the field of financial engineering primarily concerns itself with pricing and managing of derivatives (also called structured products) designed to meet specific requirements of large banks and corporations.

The pricing of such products rely on the same principles as those underlying the famous Nobel-prize winning Black-Scholes formula (Black and Scholes, 1973). While the formula is only applicable for the most simplest of derivatives (plain-vanilla Call and Put options), the underlying principle is more general. Curiously, even though the formula is named after the Nobel-winner Myron Scholes and the late Fischer Black, the name of the formula itself was given by Robert Merton, the other Nobel winner (Black, 1989).

Starting with a stochastic differential equation (SDE) for the price process of an asset, the Black and Scholes (1973) approach leads to a parabolic partial differential equation (PDE) for the price of the derivative. The Merton formulation (Merton, 1973) relies more on probabilistic ideas, and leads to the price as a mathematical expectation. These two alternative, but equivalent, approaches form the basis of much of financial engineering applications today.

While theoretically the two approaches are equivalent, in practical implementation, the PDE approach relies on Finite Difference methods and the probabilistic approach relies on Monte-Carlo simulation. This flexibility to tap into two different set of numerical techniques has proven to be of great practical importance to the industry, and has been one of the reasons behind the speed at which the products have evolved in the last two decades.

Given the complexity of structured products today almost all pricing and risk-management is based on such numerical methods. While large banks typically have their own team of 'quants' and IT developers hired to implement sophisticated Finite Difference and Monte Carlo engines, many boutique firms have spawned in the last decade who provide such specialized services/software for a fee to other banks and organizations.

Till a few years ago, however, none of that sophistication was available for use in teaching and research. Neither banks nor the boutique firms share their proprietary software, and if at all they are available, they are either prohibitively expensive or downright useless.

For the last few years, however, there is now a reliable open-source library available called QuantLib, built in C++ and also available in Python, Ruby, R and Excel among others. Originally pioneered by two Italian quants Luigi Ballabio and Ferdinando Ametrano (who continue to maintain it), over the years the library has grown substantially thanks to the contribution of many other quants and open-source developers.

This note documents our experience using QuantLib in our course on Computational Finance at the Indian Institute of Management Ahmedabad (IIMA) where we have used it both in the classroom to show pricing of fairly complex structured products and also for student's projects. The fact that it is also available (and extendable) in Python has allowed us to harness the power of C++ with the ease of iPython notebooks.

After briefly introducing the nature of the pricing problem and features of QuantLib, we discuss the case of pricing of Barrier options in some detail. It brings to fore the advantages of QuantLib compared to building stand-alone solutions.

# 2 The problem

Market for financial derivatives worldwide today is so large that the size of outstanding positions (at almost a 1000 trillion US dollars) is many times that of stock markets. A significant part of this market consists of over-the-counter structured products whose complexity varies from a European Call option to a Bermudan cross-currency Swaption.

Their sheer variety can make even the most seasoned professional dizzy. As Mark Rubinstein (Rubinstein and Reiner, 1992), a contemporary of Merton and Scholes, recalls:

"One recent advertisement promised to supply markets in 'min-max-zeros, range forwards, cylinder options, reverse forward options, quantos, zero cost collars, compound options, targets, scouts, flying hedges, moon rockets, the almost impossible to understand options' etc. and ended with the promise, 'We'll write it, you name it'"

A popular example of structured products is an 'autocallable note'. It belongs to category of products in which the capital/principal is preserved, but is designed to earn a higher interest than a typical government bond. The name autocallable comes from the fact they can be 'called back' by the seller if a certain pre-specified event occurs linked to a 'reference asset'. The reference asset could be a single stock, a basket of stocks, commodities, foreign exchange rates and/or interest rates. Such products are very popular with large corporations and banks who may have cash-flows matching or directly offsetting the event linked to the reference asset.

Another popular example is a Barrier option, which we use later to illustrate the use of QuantLib-Python. A Barrier option is a derivative whose payoff depends on whether the price of the underlying security crosses a pre-specified level (called the 'barrier') before the expiration.

The pricing problem for such derivatives typically constitutes working with a stochastic differential equation (SDE) for the price process  $(S_t)$ , like the standard Geometric Brownian Motion (GBM):

$$dS_t = rS_t dt + \sigma(t, S_t) dW_t$$

where  $W_t$  is a standard Brownian motion and the interest rate r is assumed to be a constant.

Assuming that the diffusion coefficient  $\sigma(t, S_t)$  for the Brownian motion is deterministic (as taken to be in this note), the Black-Scholes argument based on hedging (Shreve, 2007) gives the PDE for the price of the derivative  $V(t, S_t)$  as:

$$\frac{\partial V}{\partial t} + rS\frac{\partial V}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} = rV$$

with the boundary condition being the payoff at expiration of the product  $V(T, S_T)$ .

The Merton argument, on the other hand, is based on showing the existence of a 'riskneutral' probability measure and results in  $V(t, S_t)$  as the following mathematical expectation:

$$V(t, S_t) = e^{-r(T-t)} \widetilde{\mathbb{E}}[V(T, S_T)]$$

where  $\widetilde{\mathbb{E}}$  is understood to be the expectation corresponding to the risk-neutral measure.

The two approaches may look different and seem to be leading to different solutions, but that is not the case. The Feynman-Kac theorem for parabolic PDEs ensures that they are theoretically consistent (Shreve, 2007).

## 2.1 Numerical methods

For both the above approaches, the numerical methods are already well-developed in applied mathematics and probability.

In particular, for the task at hand, the most popular methods for numerically solving the PDE are Explicit and Implicit Finite Difference (FD) methods and their variants including the Crank-Nicholson and the Douglas methods. One of the more modern and most popular methods used in the industry today for multi-asset derivatives or for single assets

with stochastic volatility/interest rate is the Alternate Direction Implicit method (Duffy, 2006).

The state of the art in simulation-based methods constitutes working with Sobol sequences and low discrepancy numbers. It is also common to use variance reduction methods whenever possible, of which antithetic and control variates are the most popular (Glasserman, 2003).

Often in most practical applications it is clear which of the two approaches is better suited for a job, but in general wherever the curse of dimensionality is high (number of assets more than or equal 4), the Monte-Carlo method is preferred. When pricing products like barrier options, where the value of the underlying has to be monitored continuously or discretely, FD methods are more popular (as one can easily enforce the location of the barrier to lie on the FD grid).

Calculating first derivatives is almost natural when using the Finite Difference methods, so when it comes to calculating sensitivities for the price of the derivative ('Greeks'), the finite difference methods obviously score over Monte-Carlo, where it can be quite challenging. In practice, when using Monte-Carlo, Greeks are calculated either using path-wise differentiation or the likelihood ratio method (Duffy and Kienitz, 2009).

# 3 QuantLib

## 3.1 Background

The QuantLib open-source project was started in the year 2000 at the Italian boutique risk-management firm RiskMap (now called StatPro Italia). QuantLib website (http://www.quantlib.org) states the aim of the project as "...providing a comprehensive software framework for quantitative finance."

The first QuantLib package was released in December, 2000 under a liberal BSD license. This has allowed the banks and software companies to extend and modify the code without having to release it back. While the original developers, Luigi Ballabio and Ferdinando Ametrano, remain involved in the development and maintenance of the library, the project today has more than 150 contributors, with some of them making substantial contributions. While StatPro continues to financially support the project (which also uses it for consulting and training), growth and quality of QuantLib has been driven mainly by contributions of the open-source financial software community.

Written in C++, the library comes with more than 500 unit tests using the Boost library (Boost became a pre-requisite after July, 2004). It also makes an extensive use of SWIG (Simplified Wrapper and Interface Generator), and bindings exist for a variety of languages including Python, R, Ruby, Excel, C# and others though not all of the are equally well-developed at this stage (Python being one of the most popular and developed).

There is a host of information on the project page for any novice to get started (though it does assume a working knowledge of both C++ and quantitative finance), starting with the help on installation (http://quantlib.org/install.shtml) and video examples with

iPython notebooks (http://vimeo.com/channels/qlnotebooks) to a detailed reference manual (http://quantlib.org/reference/index.html).

Although not yet complete as on date, Luigi Ballabio has written a companion book (Ballabio, 2015) describing the more important classes (including modeling financial instruments and term structures) and pricing frameworks used in Monte-Carlo and Finite Differences methods. He also maintains a fairly regular blog documenting its development and features (http://www.implementingquantlib.com).

In Appendix A, we provide installation and set-up instructions for QuantLib and QuantLib-Python for both Linux (Ubuntu) and Windows operating systems.

## 3.2 Features

Other than the fact that it is free and extremely well-designed, QuantLib is particularly attractive because it allows one to focus on important pricing problems rather than worrying about some basic, but often extremely tedious classes like Date and Interpolation. There are also no surprises as nothing is a black-box - all source code is available to check the exact implementation details of the algorithms and the pricing engines. As with all mature open-source projects, with expert contributors and administrators, QuantLib comes with a reliability and ease of bug-fixing making it perfectly suited for applications in teaching, research and small firms (Firth, 2004).

The way it is developed, QuantLib is completely object-oriented and makes extensive use of design patterns. Even if someone is not developing models, it is a good example to learn the use of design patterns when building a financial library.

All the standard object-oriented design features are part of QuantLib, including polymorphism (methods are inherited by subclasses), enumeration (list of fixed parameters), portability (platform neutral), serialization (save objects as XML) and coercion (automatic data-type conversion).

In terms of financial applications, QuantLib not only includes classes for market conventions and yield curve models but also comes with low-discrepancy sequences and solvers for PDEs with a large choice of alternative algorithms and exotic payoffs. With modern multi-core processors, QuantLib also allows multithreading via OpenMP (under development).

## 3.3 Structure of QuantLib: Important classes

Price of any derivative, be it a plain-vanilla option or a complex structured product, depends on the following inputs:

- Price of the underlying securities as on date of pricing and their feeds
- Term structure of interest rates, volatility, inflation and default probabilities
- Cash flows (including coupons and dividends) from the instrument
- Stochastic process for the underlying

• Pricing engine (the numerical method used for pricing)

Now if one is pricing a stand-alone product for a 'student project', one could just code the above elements in a single monolithic program to find the price, and it is not altogether necessary to use an object-oriented approach like QuantLib.

In a large financial institution or a hedge fund, however, derivatives are part of a larger portfolio and then this monolithic approach quickly becomes inefficient and impractical.

The beauty of QuantLib is that in its spirit and scope, it is very similar to financial library that one finds in a large bank. It then becomes natural to use QuantLib for computational finance even in the classroom, as it helps students come up to speed with the state-of-the-art quickly. This not only helps make the curriculum more relevant, it also helps attract the best and most interested students to the class (and the institution).

While QuantLib reference manual is the source to refer to understand the library, next we describe the important classes that get used in much of pricing.

#### 3.3.1 The Instrument class

The Instrument class is designed keeping in mind that as the value of the underlying security changes, so does the value of a instrument. It is then required to maintain 'links' so that when called they would access most current values of the underlying. At the same time 'caching' is desirable - that is the value of the instruments should be recalculated only when one or more of the input values have changed.

This is operationalized via the Observer design pattern in QuantLib, where instrument plays the role of the 'observer' and inputs that of 'observables' (for details, see Ballabio (2015), Chapter 2).

#### 3.3.2 The TermStructure class

It is only in the textbook version of the Black-Scholes that interest rates and volatility are treated as constants. In the messy world of central bank interventions and market volatility, both are time-varying.

The **TermStructure** class is responsible for constructing time-varying objects for all such variables. In particular, its job is to:

- Keep track of its 'own' (reference) date and calculate the appropriate future date if reference date is different from the evaluation date (by using number of business days forward)
- Convert dates to times (say, when converting discount factors to zero-yields)
- Check whether a given date/time belongs to the domain covered by the term structure (maximum allowed date/range, esp. when fitting volatility term structures)

With reference dates described in the **TermStructure** class, there exist specific classes to do the job of specifying yield curve, volatility, inflation and so on.

- The YieldTermStructure class provides the means to specify discount-based yield curves. To make it easier to handle zero-rates and forward rates directly, the Adapter pattern is used to convert discount-based interface in the YieldTermStructure to zero rates (ZeroYieldStructure) and forward rates (ForwardRateStructure).
- To model risky term structures, the DefaultProbabilityTermStructure class is used. Again, the Adapter pattern is used to go from default probabilities to survival probability (SurvivalProbabilityTermStructure), default density (DefaultDensityStructure) and hazard rates (HazardRateStructure).
- Even though the nature of volatility term structure differs across asset classes, there are enough commonalities (esp. in relation to 'time' and 'strikes') for QuantLib to have a generic VolatilityTermStructure inherting from the TermStructure class to capture volatility surface. Within the VolatilityTermStructure class, the equity and FX volatility are specified using the BlackVolTermStructure and interest rate volatility surface using the CapFloorTermVolatilityStructure
- The InflationTermStructure class is used to specify time-varying inflation.

Although not a 'base' class on its own (and more of a utility), as far as the TermStructure class is concerned, the importance of the Date class cannot be over-emphasized. Model-ing fixed-income securities issued by companies in different countries necessitates work-ing with calendars, and handling holidays can be a nightmare when working with a multi-currency/country portfolio.

For historical reasons there are conventions in each country on how number of days in a month are counted, and these are again of particular importance when working with fixed-income securities like swaps. Setting up a calendar in QuantLib is as easy as calling myCalendar = TARGET(), for example. The calendars are customizable so one can easily add/remove specific holidays from the object via addHoliday and removeHoliday procedures.

The class is described in detail in Ballabio (2015), but the way the Date class is constructed allows one to do usual addition/subtraction of dates as one would in Microsoft Excel. Other C++ convenient operators ++, -, +=, -= can also be used. It class also contains some very useful functions for checking for things like a leap year, end of the month, day of the week etc.

#### 3.3.3 The Payoff and the Exercise class

All derivatives require specification of a payoff at expiration/early close-out. In QuantLib, the payoffs are derived from the Payoff class whose descendants include, among others, PlainVanillaPayoff (which could be set to Call or Put by means of a switch), AssetOrNothingPayoff and many others. Its use is illustrated later in our example.

For derivatives with early-exercise feature, a crucial property is the exercise possibilities available. In QuantLib this is operationalized using the Exercise base class, which depending on exercise choices available, can be accessed as one of EuropeanExercise (only at expiration) or BermudanExercise (on a set of discrete dates prior to expiration) or AmericanExercise (any time before expiration) classes.

#### 3.3.4 The StochasticProcess class

All derivatives pricing begins with the assumption of an appropriate stochastic process for the underlying and volatility. For example, Black-Scholes model begins with a GBM for the stock price, and the Heston model (Heston, 1993) builds on it by adding a square root process for instantaneous variance.

In QuantLib, stochastic processes are specified using the StochasticProcess Observer design pattern. The StochasticProcess class descends into a discretization class which handles how the process is passed into the pricing engine, of which the EulerDiscretization is the most important which implements the evolution of the un-

derlying as:

$$S(t+\Delta t)=rS(t)\Delta t+\sigma(t,S(t))\Delta W_t$$

The **StochasticProcess** base class further descends into some standard stochastic processes including:

- The GeneralizedBlacksScholesProcess class: This base class represents the standard BlackScholesProcess class for modeling stocks without dividend, the general BlackScholesMertonProcess class and the GarmanKohlagenProcess class for currencies
- The OrnsteinUhlenbeckProcess class: This base class for square-root stochastic processes is very commonly used to model interest rates and stochastic volatility
- The HestonProcess and the BatesProcess classes: These allow for handling standard stochastic volatility models of the Heston-type.

## 3.3.5 The PricingEngine class

An instrument can be priced using any of the different pricing engines available in the library and with different objectives in mind. For example a Call option may need be priced for its own sake, or to derive implied volatilities or for calibrating a stochastic volatility model. And any of these may need to be done via FD or Monte-Carlo methods.

In QuantLib this is operationalized using the PricingEngine class which is modeled as the Strategy pattern, in which instruments takes an object encapsulating the computation to be performed. This allows the instrument to be priced using any of the various engines available in the library.

For reasons of implementation, most pricing engines do not descend from the **PricingEngine** class, but from a generic subclass called **GenericEngine** (for details, see Ballabio (2015), Chapter 2). Currently the pricing engines are available for Asian, Barrier, Basket, Cap/floor, Cliquet, Forward, Quanto, Swaption and Vanilla option engines. For most engines, all three procedures, including Analytic, FD and Monte-Carlo methods are available.

#### 3.3.6 Other classes

Another important class, especially for pricing fixed-income derivatives is the generic CashFlow class. Its interface ranges from monitoring dates and the associated cash flow (the SimpleCashFlow class) to keeping track of accruing cash flows with a given day-count convention (the Coupon class).

Within the Coupon base class exist the FixedRateCoupon and the FloatingRateCoupon classes implementing cash flows for different kind of bonds (leg of swaps). Since our example is not from fixed-income derivatives, we would not spend more time on the CashFlow class here. For more details, the reader is referred to Ballabio (2015), Chapter 4.

Although there are other important classes implementing processes like random number and path generation (RandomSequenceGenerator and Path classes), calibration (the CalibrationHelper and CalibratedModel classes) and models like 'trees' (the Lattice and DiscretizedAsset classes), again, since they aren't discussed in our example we leave the details for these to the relevant chapters of Ballabio (2015).

Other than the Date class, QuantLib also contains a set of utilities for data types, market quotes, indexes (like stock index), error reporting and select math-related class including interpolation, one-dimensional solvers, optimizers and statistics.

Hopefully this gives a big-picture overview of the library and what it can achieve. Now we move on to describe the Barrier option case-study.

# 4 Example: Pricing Barrier option using FD

Since the barrier option is a challenging problem for numerical valuation methods like FD and Monte-Carlo, we choose to value a Down-and-Out Call option using FD method implemented in QuantLib. The correct value of this option is available as a closed-form formula (Haug, 2007), and therefore we can easily compute the pricing error of the numerical method.

## 4.1 Product: Barrier option

A Barrier option (colloquially referred to simply as 'barriers') is a derivative whose payoff depends on whether the price of the underlying security crosses a pre-specified level before the expiration. It comes in two basic varieties:

- Knock-in: The option gives a payoff only if the barrier is breached before expiration
- Knock-out: The option expires worthless if the barrier is breached before expiration

Barriers are also typically categorized in relation to the current value of the underlying. So if the barrier level of a knock-out option is set to a value below (above) the current value of the asset, it is referred to as a 'Down-and-Out' ('Up-and-Out') option. The 'in' options are defined similarly. In plain-vanilla barriers, the payoff at expiration could be either of a Call or a Put variety. Putting all variants together, then, there exist 8 different combinations of barrier options. To mathematically represent the payoff of a barrier option, it is convenient to call  $m_T$  and  $M_T$  as the minimum and maximum value respectively of the asset between the evaluation (t) and expiration (T) date as:

$$m_T = \min_{0 \le t \le T} S_t$$
$$M_T = \max_{0 \le t \le T} S_t$$

Given a barrier level  $B_d$ , the payoff of a Down-and-Out Call option, for example, is then written as:

Down-and-out Call payoff = 
$$V_T = \max(S_T - K, 0) \times \mathbb{I}[m_T > B_d]$$

where  $\mathbb{I}[m_T > B_d]$  represents an indicator variable which takes the value 1 if the minimum value of the asset before expiration lies above the barrier.

Similarly for an Up-and-In Put option with barrier  $B_u$ , the payoff is written as:

Up-and-in Put payoff = 
$$V_T = \max(K - S_T, 0) \times \mathbb{I}[M_T > B_u]$$

where  $\mathbb{I}[M_T > B_u]$  represents an indicator variable which takes the value 1 if the maximum value of the asset before expiration crosses the barrier  $B_u$ .

Assuming that the barrier option is attainable, self-financing condition and Ito's Lemma leads to the standard Black-Scholes PDE for value of barrier  $V_t$ :

$$\frac{\partial V}{\partial t} + rS\frac{\partial V}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} = rV$$

with the following boundary conditions:

$$V(t, B_d) = 0, \qquad 0 \le t \le T$$
$$V(T, S_t) = \max(S_T - K, 0), \qquad S_t > B_d$$

Other variants of barrier options are handled similarly, except that the math (and implementation) is easier for pricing 'out' options. It is then standard to use the fact that the sum of an 'out' and 'in' option must be the same as the price of an equivalent plainvanilla option. The price of an 'in' option is then derived as a difference between the price of an equivalent plain-vanilla and an 'out' option.

#### 4.2 Method: Alternative FD schemes

The FD method begins with discretizing the three partial derivatives in the Black-Scholes PDE, solving and propagating 'back' from the boundary condition at expiration to the evaluation date.

While discretizing the partial derivatives, whether one does backward or forward difference turns out to be important. The Explicit FD scheme uses backward difference in t, and the Implicit FD scheme uses the forward difference in t.

Denoting the value of the option at each point on the FD grid (see Figure 1), the notation below follows from Wilmott (2006), Chapter 77 as:

$$V_i^k = V(i\delta S, T - k\delta t)$$

where the grid is thought to be made of points in asset values  $(S = i\delta S)$  and times  $(t = T - k\delta t)$ , with time counted 'backwards' from expiration to evaluation date. For more on the choice of grid-size etc., the reader is referred to Duffy (2006) or Wilmott (2006).

In particular, in the Explicit FD scheme the space partial derivatives are discretized as:

$$\frac{\partial V}{\partial S} \approx \frac{V_{i+1}^k - V_{i-1}^k}{2\delta S}$$
$$\frac{\partial^2 V}{\partial S^2} \approx \frac{V_{i+1}^k - V_i^k + V_{i-1}^k}{\delta S^2}$$

and in the Implicit FD scheme they are discretized as:

$$\frac{\partial V}{\partial S} \approx \frac{V_{i+1}^{k+1} - V_{i-1}^{k+1}}{2\delta S}$$
$$\frac{\partial^2 V}{\partial S^2} \approx \frac{V_{i+1}^{k+1} - V_i^{k+1} + V_{i-1}^{k+1}}{\delta S^2}$$

The time partial derivative is implemented in the same way in both schemes as:

$$\frac{\partial V}{\partial t} \approx \frac{V_i^k - V_i^{k+1}}{\delta t}$$

The Douglas scheme is a sort of weighted-average of the Implicit and Explicit FD schemes, and is implemented as:

$$\begin{split} &\frac{\partial V}{\partial S}\approx \theta\Big(\frac{V_{i+1}^k-V_{i-1}^k}{2\delta S}\Big)+(1-\theta)\Big(\frac{V_{i+1}^{k+1}-V_{i-1}^{k+1}}{2\delta S}\Big)\\ &\frac{\partial^2 V}{\partial S^2}\approx \theta\Big(\frac{V_{i+1}^k-V_i^k+V_{i-1}^k}{\delta S^2}\Big)+(1-\theta)\Big(\frac{V_{i+1}^{k+1}-V_i^{k+1}+V_{i-1}^{k+1}}{\delta S^2}\Big) \end{split}$$

Note that  $\theta = 1$  gives back the Explicit scheme, and  $\theta = 1$  gives back the Implicit scheme. With  $\theta = 1/2$ , one gets the scheme known as Crank-Nicholson method.

Borrowing the pictures from Wilmott (2006), the comparison between the schemes can be represented using a lattice diagram as in Figure 1 below.

Given the time-step of  $\delta t$  and space-step of  $\delta S$ , the error in both Explicit and Implicit FD scheme is of the order  $\mathcal{O}(\delta t, \delta S^2)$ , so it decreases like  $\delta t$ . While there is little difference in computational effort required to implement Implicit and Crank-Nicholson schemes, in the latter the error decreases as  $\delta t^2$ , so is always preferred.



Figure 1: Explicit (top), Implicit (middle) and Crank-Nicholson/Douglas FD (bottom) schemes (Source: Wilmott (2006), Chapters 77 and 78)

One can do even better than Crank-Nicholson and select  $\theta$  optimally in the Douglas scheme, set to:

$$\theta = \frac{1}{2} - \frac{\delta S^2}{12\delta t}$$

The optimal  $\theta$  is obtained by manipulating the fourth-order difference equation to eliminate the fourth-order terms leading to significant error reduction. The resulting error rate is  $\mathcal{O}(\delta t^2, \delta S^4)$ . For more details on all the schemes and more, see Duffy (2006) and Wilmott (2006).

# 5 Implementation

By now we have all the necessary set-up to talk about implementation of our example in QuantLib-Python.

## 5.1 QuantLib to QuantLib-Python

Once QuantLib and QuantLib-Python have been installed, the single line:

```
from QuantLib import *
```

at the beginning of a Python script is sufficient to provide access to all the functions in the QuantLib library. It is not necessary to know anything about the C++ language at all. The Python programmer works with Python variables and calls Python functions. The QuantLib-Python module automatically translates these into the appropriate C++ function and converts the C++ objects into Python objects. To the programmer, it is as if the entire QuantLib library had been written in Python instead of C++.

All this magic is accomplished by the SWIG software which creates wrapper code that converts between Python and C++ data types before and after calling a C++ function. In the simplest situation the wrapper code does only three things:

- 1. All the input arguments to the function are converted from Python data types to C++ data types
- 2. The C++ function is called with the arguments provided as C++ data types
- 3. The return value from C++ function is converted from C++ data types to Python data types which the Python programmer can use.

In simple cases, SWIG can parse the C++ source code and generate the wrapper code without too much manual intervention. In the case of a complex software like QuantLib, it is necessary to give SWIG considerable guidance on how to build the wrapper. This is done using interface files (which usually have the .i extension). Moreover, where the C++ code uses templates, each instance of the template must be wrapped separately, and often the interface file instructs SWIG to wrap only the most common instances of the template.

The QuantLib library comes with a set of interface files that build wrappers for the most important QuantLib functions. In most cases, therefore, the programmer does not have to

worry about the interface files at all. During the installation process, the predefined interface files would be used to build the wrapper files and create the QuantLib Python module. The programmer can simply add the line "from QuantLib import \*" to the Python code and not worry about C++ and SWIG at all.

In some cases, the predefined interface files may not provide access to a QuantLib function that is required for the specific task at hand. In our case, the predefined interface files do not provide access to the FD pricing engine for barrier options. To use this engine in Python code, it is necessary to modify the interface file and instruct SWIG to build a wrapper for this engine also. The patch file in Appendix E shows the lines to be added to the predefined options.i file to achieve this. Rebuilding the wrappers and the QuantLib Python module using this modified file allows the Python programmer to use the FD pricing engine for barrier options in Python code.

In our experience, the predefined interface files are quite comprehensive and we have encountered only a couple of instances requiring modification of the interface files and rebuilding the module.

## 5.2 The code: fdbarrier.py

The listing in Appendix D illustrates how QuantLib is used in Python. This example used Python 3, but Python 2.7 would work equally well.

Line 3 in the listing (from QuantLib import \*) has already been discussed; the import in line 4 is for plotting. Lines 5 to 9 define the inputs for the barrier option and are pure Python code except that they refer to two constants (Barrier.DownOut and Option.Call) defined in QuantLib. We are defining a Down-and-Out Call option with a barrier at 80 and no rebate. The current market price of the underlying is 100, the strike price of the option is 105, the risk free rate is 5%, the volatility is 20% and the maturity is 1 year.

Lines 10 and 11 define the different grid spacing parameters that we will use for the finite difference method. The number of grid points for the space (asset price) dimension and time dimension range from 5 to 5000.

Lines 12 and 13 set the maturity date. Settings.instance().evaluationDate is a QuantLib global variable that defines the date on which the evaluation is done. We add the requisite number of months to this to get the maturity date.

Lines 14 to 19 set up the stochastic process. This function takes the following arguments: the current market price of the underlying, the dividend yield, the risk free rate, and the volatility. The last three arguments are converted into instances of the term structure class discussed in Section 3.3.2 above. The first argument has to be converted into a Quote-Handle which is a QuantLib class used for market quotes.

Lines 20 to 22 set up the barrier option itself. It may be observed that the last two arguments to this function use the payoff class and the exercise class discussed in Section 3.3.3 above. In lines 23 and 24, we find the true (analytic) value of the option by first setting the analytic pricing engine and then computing the NPV of the option.

Lines 25 to 33 perform the FD valuation of the barrier option for two different sets of the space and time grids. First, we keep the time grid fixed at the maximum value of 5000

and vary the space grid from 5 to 5000. The pricing engine is set to the Finite Difference engine, the option is valued and the pricing error (relative to the true analytic value) is calculated and stored in uErrors. Second, we keep the space grid fixed at the maximum value of 5000 and vary the time grid from 5 to 5000. The pricing error is calculated and stored in tErrors.

At the end, we plot the results on a log-log scale (line 34); the rest of the code sets up the titles and legend for the plot.

#### 5.3 Results and extensions



Figure 2: Dependence of Pricing Accuracy on Number of Asset Price Grid Points and Time Grid Points

The graph produced by the code in Section 5.2 is shown in Figure 2. This graph can be used to advance several pedagogical purposes. First the red line (dependence of pricing accuracy on asset price grid) conforms to the theoretical prediction of a straight line in a log-log plot. Second the initial segment of the blue line (dependence of pricing accuracy on time grid) is also a straight line in accordance with theoretical predictions. But the later segment of the curve is almost flat. This is again generally observed in practice: it is usually optimal to have significantly finer grid on the asset price than on the time dimension. Increasing the time grid points without simultaneously increasing asset grid points to a significantly larger value is often futile.

One question that will arise at the end is what is the point of doing valuation using FD of an instrument for which there is an analytic pricing formula. The answer is that the FD method will work under alternative assumptions where there is no analytic formula. For example, if change the stochastic process from the GBM or Black-Scholes process to a stochastic volatility or Heston-type process, there is no analytic formula. But the FD method will work equally well even in this case. The results of the GBM case can guide us in choosing the appropriate number of grid points to compute the option value.

# 6 Conclusion

This note has introduced QuantLib and QuantLib-Python for pricing derivative securities in practice. Given the sophistication of pricing models used at financial institutions, reliance on numerical methods is unavoidable. There is currently no open-source computing library (other than the OpenGamma<sup>1</sup> platform perhaps) which allows capturing real-world pricing issues sitting in the ivory tower. While the set-up cost may be high, the time learning the structure of QuantLib is well worth it.

Once the framework of QuantLib is clear, one need not even know C++. Thanks to SWIG, one can simply work with QuantLib-Python, and a programmer need only know Python well. Even though the predefined interface of QuantLib with Python via SWIG files may not be complete, in our experience it is comprehensive enough that requiring modification of the interface and rebuilding QuantLib-Python is quite infrequent.

At the risk of being overly-optimistic, hopefully this note will act as a nudge for some financial engineers (and computational finance instructors) out there to ditch Microsoft Excel and other proprietary software in favour of QuantLib.

<sup>&</sup>lt;sup>1</sup>See http://www.opengamma.com/opengamma-platform. OpenGamma seems to be primarily designed for margining and market-risk management applications however.

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# A Box: Installation Instructions for QuantLib and QuantLib-Python

For QuantLib-Python to work it is necessary to first have QuantLib working. After listing out the common prerequisites, we provide step-by-step instructions separately for Ubuntu and Windows.

#### Prerequisites

- Operating system: Ubuntu 12.04 and above (preferably the LTS 12.04 or 14.04) and Windows 7 and above. They should well work for other Linux distributions, but we haven't tested it.
- Free space and RAM: At least 64 GB free space (preferably 250 GB and above), and 1 GB of RAM (preferably 4 GB and above) is recommended. This is to ensure that the system has access to adequate virtual memory.
- For installing in Windows, a working C++ environment is required. For the purposes of instructions here (meant for a novice), it is recommended that the user install the free Microsoft Visual Express Desktop 2013 edition which comes with Visual C++ 12 (MSVC12). It is equally easy to install in Windows on Cygwin, or using MinGW, but then a user familiar with those would probably not need this userguide.
- QuantLib-Python requires a working Python environment, and we recommend working with the Anaconda meta-package. Instructions for both Ubuntu and Windows are available from the Anaconda install page (docs.continuum.io/anaconda/install. html). For our purpose here, Windows users should work with the 32-bit version of Anaconda. It is recommended to install Anaconda with all the default settings, and update it by running conda update conda and conda update anaconda. Again, it does not matter if one installs Anaconda-2.x (with Python 2.7) or Anaconda-3.x (with Python 3), but it is recommended that the user install the Python 3 version, as that's where the Python language seems to be headed.
- Additional ingredients include Boost C++ libraries and (after that) SWIG, instructions for which are different for Ubuntu and Windows and is provided below.
- One should download the same version of QuantLib and QuantLib-Python (http: //sourceforge.net/projects/quantlib/files/QuantLib/1.6/).

QuantLib-Python requires a working QuantLib, so the user should follow the instructions in the order as given below.

## Installing in Ubuntu

For Ubuntu it is recommended not to install QuantLib from the synaptic package manager as Ubuntu repositories do not contain the latest version.

• First step is installing the Boost C++ libraries, and in Ubuntu they are available from the repositories (package name libboost-all-dev)

- After installing Boost, user should install SWIG, also available from the repositories (package name swig)
- Instructions for installing QuantLib for Ubuntu are available from the QuantLib project page at quantlib.org/install/linux.shtml. Before proceeding further, the user should ensure that examples given on the QuantLib page are working and not giving any errors.
- After installing QuantLib, QuantLib-Python requires running the following steps (in that order):
  - cd\path\to\QuantLib-SWIG-1.6\Python
  - python setup.py wrap
  - python setup.py build
  - python setup.py test
  - sudo python setup.py install

Although there is no problem in installing Boost and SWIG from their version available in repositories for QuantLib-Python, the versions available there are somewhat old compared to the latest version. For users with some experience in installing packages in Linux, it is definitely recommended to work with the latest versions by installing them from source.

If user-defined .i SWIG files have been added (or existing files have been modified), to ensure that they are available, the last four steps (from wrap to install) must be repeated.

#### Installing in Windows

If the Windows user is working with a free version of Visual Studio, it should be kept in mind to install the 32-bit versions of both Anaconda and Boost as free Visual Studio lacks the necessary toolkit for building QuantLib for 64-bit (there are ways to make the free version work in 64-bit, but they are not guaranteed to be replicated universally). Those with access to the professional Visual Studio environment may choose 64-bit for every-thing.

Throughout \path\to\someplace represents the directory of someplace. For example, if boost\_1\_58\_0 is installed in C:\boost, \path\to\boost\_1\_58\_0 should be taken to mean C:\boost\boost\_1\_58\_0.

- For Windows, pre-packaged binaries for boost are available (latest version 1.58.0) for specific versions of MSVC (in our case MSVC12) from its sourceforge page. User should download the executable for the 32-bit architecture (http://sourceforge.net/projects/boost/files/boost-binaries/1.58.0/).
- After installing boost, user should install SWIG. Pre-packaged binaries for SWIG for Windows are available from the source page (www.swig.org/download.html), and it is enough to extract the SWIG zip file (latest version swigwin-3.0.5) in a convenient folder.
- All the commands below assume that we are working in Visual Studio command prompt (i.e. all the relevant Visual Studio related environment variables have been

set). It can be launched from the Start menu (in Windows 7) or from Apps (in Windows 8). Alternatively, one can find how to locate it at https://msdn.microsoft.com/en-us/library/ms229859(v=vs.110).aspx

- QuantLib in Windows can directly be installed in three simple steps after launching the Visual Studio command prompt (mind the gaps):
  - $\operatorname{cd} \operatorname{bath}toQuantLib-1.6$
  - set myboost=\path\to\boost
  - msbuild /p:AdditionalLibPaths="%myboost\lib32-msvc-12.0"/p:Configuration= Release/p:Platform=Win32QuantLib\_vc12.sln
- The last step above may take a while (on an Intel i5, Windows 7 machine with 4 GB RAM it took almost 45 minutes), so it is advisable to have a couple of hours at hand when sitting down to install QuantLib.
- Installing QuantLib-Python in Windows requires some extra settings (again, after launching the Visual Studio command prompt):
  - cd\path\to\QuantLib-SWIG-1.6\Python
  - At the command prompt set the following (mind the gaps):
    - \* set PATH=\path\to\Anaconda3;\path\to\Anaconda3\scripts;\path\to\
      swigwin-3.0.5;\path\to\QuantLib-1.6\lib;%PATH%
    - \* set QL\_DIR=\path\to\QuantLib-1.6
    - \* set VS100COMNTOOLS=%VS120COMNTOOLS%
    - \* set INCLUDE=\path\to\boost\_1\_58\_0;%INCLUDE%
    - \* set LIB=\path\to\boost\lib32-msvc-12.0;%LIB%
    - \* echo [build] > setup.cfg && echo compiler=msvc >> setup.cfg
- The last few steps are identical to that in Ubuntu:
  - cd\path\to\QuantLib-SWIG-1.6\Python
  - python setup.py wrap
  - python setup.py build
  - python setup.py test
  - python setup.py install

If user-defined .i SWIG files have been added (or existing files have been modified), to ensure that they are available, one needs to additionally go through the 'set' commands before running the last four steps (from wrap to install).

# B Box: SWIG options.patch file

```
2015-01-16 16:22:45.000000000 +0530
--- options.i
+++ modified-options.i
                             2015-02-23 13:30:23.096148000 +0530
@@ -1099,6 +1099,63 @@ class MCBarrierEnginePtr : public boost:
    }
 };
+
+%{
+using QuantLib::FdmSchemeDesc;
+%}
+
+struct FdmSchemeDesc {
  enum FdmSchemeType { HundsdorferType, DouglasType,
+
+
                       CraigSneydType, ModifiedCraigSneydType,
                       ImplicitEulerType, ExplicitEulerType };
+
+
+
  FdmSchemeDesc(FdmSchemeType type, Real theta, Real mu);
+
+
  const FdmSchemeType type;
  const Real theta, mu;
+
+
+ // some default scheme descriptions
+ static FdmSchemeDesc Douglas();
+ static FdmSchemeDesc ImplicitEuler();
+ static FdmSchemeDesc ExplicitEuler();
+ static FdmSchemeDesc CraigSneyd();
+ static FdmSchemeDesc ModifiedCraigSneyd();
+ static FdmSchemeDesc Hundsdorfer();
+ static FdmSchemeDesc ModifiedHundsdorfer();
+};
+
+%{
+using QuantLib::FdBlackScholesBarrierEngine;
+typedef boost::shared_ptr<PricingEngine> FdBlackScholesBarrierEnginePtr;
+%}
+
+%rename(FdBlackScholesBarrierEngine) FdBlackScholesBarrierEnginePtr;
+class FdBlackScholesBarrierEnginePtr : public boost::shared_ptr<PricingEngine> {
  public:
+
+
    %extend {
        FdBlackScholesBarrierEnginePtr(const GeneralizedBlackScholesProcessPtr& process,
+
          Size tGrid = 100, Size xGrid = 100, Size dampingSteps = 0,
+
          const FdmSchemeDesc& schemeDesc = FdmSchemeDesc::Douglas(),
+
          bool localVol = false,
+
```

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```
Real illegalLocalVolOverwrite = -Null<Real>()) {
+
+
           boost::shared_ptr<GeneralizedBlackScholesProcess> bsProcess =
                boost::dynamic_pointer_cast<GeneralizedBlackScholesProcess>(
+
                                                               process);
+
           QL_REQUIRE(bsProcess, "Black-Scholes process required");
+
           return new FdBlackScholesBarrierEnginePtr(
+
                          new FdBlackScholesBarrierEngine(bsProcess,
+
                                           tGrid, xGrid, dampingSteps,
+
+
                                           schemeDesc, localVol,
+
                                           illegalLocalVolOverwrite));
        }
+
+ }
+};
+
+
+
+
%{
using QuantLib::QuantoEngine;
using QuantLib::ForwardVanillaEngine;
```

# C Box: FD Barrier Python Code

```
1 #!/usr/bin/env python3
2 # Requires QuantLib-SWIG with modified options.i
3 from QuantLib import *
4 import matplotlib.pyplot as plt
5 barrier, barrierType, optionType, \setminus
       rebate = (80.0, Barrier.DownOut, Option.Call, 0.0)
6
7
  underlying, strike, rf, sigma, maturity, \
       divYield = (100, 105, 5e-2, 20e-2, 1.0, 0.0)
8
9\, # maturity is in years & must correspond to an integral number of months
10 Grids = (5, 10, 25, 50, 100, 1000, 5000)
11 \text{ maxG} = \text{Grids}[-1]
12 today = Settings.instance().evaluationDate
13 maturity_date = today + int(maturity * 12)
14
   process = BlackScholesMertonProcess(
       QuoteHandle(SimpleQuote(underlying)),
15
       YieldTermStructureHandle(FlatForward(today, divYield, Thirty360())),
16
17
       YieldTermStructureHandle(FlatForward(today, rf, Thirty360())),
18
       BlackVolTermStructureHandle(BlackConstantVol(
           today, NullCalendar(), sigma, Thirty360()))
19
20
   option = BarrierOption(barrierType, barrier, rebate,
21
                           PlainVanillaPayoff(optionType, strike),
22
                           EuropeanExercise(maturity_date))
23 option.setPricingEngine(AnalyticBarrierEngine(process))
24 trueValue = option.NPV()
25 uErrors = []
26 \text{ tErrors} = []
27
  for Grid in Grids:
28
       option.setPricingEngine(FdBlackScholesBarrierEngine (
29
           process, maxG, Grid))
30
       uErrors.append(abs(option.NPV()/trueValue-1))
31
       option.setPricingEngine(FdBlackScholesBarrierEngine (
32
           process, Grid, maxG))
33
       tErrors.append(abs(option.NPV()/trueValue-1))
34 plt.loglog(Grids, uErrors, 'r-', Grids, tErrors, 'b--')
  plt.xlabel('No of Grid Points (Log Scale)')
35
36 plt.ylabel('Relative Error (Log Scale)')
37 plt.legend(['Asset Grid Points', 'Time Grid Points'])
38 plt.title('Increasing Asset or Time Grid Keeping the Other Grid at ' + str(maxG))
39 plt.show()
```

## D Python-QuantLib code for Barrier option: fdbarrier.py

```
1 #!/usr/bin/env python3
2 # Requires QuantLib-SWIG with modified options.i
3 from QuantLib import *
4 import matplotlib.pyplot as plt
5 barrier, barrierType, optionType, \setminus
       rebate = (80.0, Barrier.DownOut, Option.Call, 0.0)
6
7
  underlying, strike, rf, sigma, maturity, \setminus
       divYield = (100, 105, 5e-2, 20e-2, 1.0, 0.0)
8
9\, # maturity is in years & must correspond to an integral number of months
10 Grids = (5, 10, 25, 50, 100, 1000, 5000)
11 \text{ maxG} = \text{Grids}[-1]
12 today = Settings.instance().evaluationDate
13 maturity_date = today + int(maturity * 12)
14 process = BlackScholesMertonProcess(
       QuoteHandle(SimpleQuote(underlying)),
15
       YieldTermStructureHandle(FlatForward(today, divYield, Thirty360())),
16
17
       YieldTermStructureHandle(FlatForward(today, rf, Thirty360())),
18
       BlackVolTermStructureHandle(BlackConstantVol(
           today, NullCalendar(), sigma, Thirty360()))
19
20
   option = BarrierOption(barrierType, barrier, rebate,
21
                           PlainVanillaPayoff(optionType, strike),
22
                           EuropeanExercise(maturity_date))
23 option.setPricingEngine(AnalyticBarrierEngine(process))
24 trueValue = option.NPV()
25 uErrors = []
26 \text{ tErrors} = []
27
   for Grid in Grids:
28
       option.setPricingEngine(FdBlackScholesBarrierEngine (
29
           process, maxG, Grid))
30
       uErrors.append(abs(option.NPV()/trueValue-1))
31
       option.setPricingEngine(FdBlackScholesBarrierEngine (
32
           process, Grid, maxG))
33
       tErrors.append(abs(option.NPV()/trueValue-1))
34 plt.loglog(Grids, uErrors, 'r-', Grids, tErrors, 'b--')
  plt.xlabel('No of Grid Points (Log Scale)')
35
36 plt.ylabel('Relative Error (Log Scale)')
37 plt.legend(['Asset Grid Points', 'Time Grid Points'])
38 plt.title('Increasing Asset or Time Grid Keeping the Other Grid at ' + str(maxG))
39 plt.show()
```

## E options.patch

```
2015-01-16 16:22:45.00000000 +0530
--- options.i
+++ modified-options.i
                             2015-02-23 13:30:23.096148000 +0530
@@ -1099,6 +1099,63 @@ class MCBarrierEnginePtr : public boost:
    }
 };
+
+%{
+using QuantLib::FdmSchemeDesc;
+%}
+
+struct FdmSchemeDesc {
  enum FdmSchemeType { HundsdorferType, DouglasType,
+
+
                       CraigSneydType, ModifiedCraigSneydType,
                       ImplicitEulerType, ExplicitEulerType };
+
+
+
  FdmSchemeDesc(FdmSchemeType type, Real theta, Real mu);
+
+
  const FdmSchemeType type;
  const Real theta, mu;
+
+
+ // some default scheme descriptions
+ static FdmSchemeDesc Douglas();
+ static FdmSchemeDesc ImplicitEuler();
+ static FdmSchemeDesc ExplicitEuler();
+ static FdmSchemeDesc CraigSneyd();
+ static FdmSchemeDesc ModifiedCraigSneyd();
+ static FdmSchemeDesc Hundsdorfer();
+ static FdmSchemeDesc ModifiedHundsdorfer();
+};
+
+%{
+using QuantLib::FdBlackScholesBarrierEngine;
+typedef boost::shared_ptr<PricingEngine> FdBlackScholesBarrierEnginePtr;
+%}
+
+%rename(FdBlackScholesBarrierEngine) FdBlackScholesBarrierEnginePtr;
+class FdBlackScholesBarrierEnginePtr : public boost::shared_ptr<PricingEngine> {
+ public:
+
    %extend {
        FdBlackScholesBarrierEnginePtr(const GeneralizedBlackScholesProcessPtr& process,
+
          Size tGrid = 100, Size xGrid = 100, Size dampingSteps = 0,
+
          const FdmSchemeDesc& schemeDesc = FdmSchemeDesc::Douglas(),
+
          bool localVol = false,
+
```

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```
Real illegalLocalVolOverwrite = -Null<Real>()) {
+
+
           boost::shared_ptr<GeneralizedBlackScholesProcess> bsProcess =
                boost::dynamic_pointer_cast<GeneralizedBlackScholesProcess>(
+
                                                               process);
+
           QL_REQUIRE(bsProcess, "Black-Scholes process required");
+
           return new FdBlackScholesBarrierEnginePtr(
+
                          new FdBlackScholesBarrierEngine(bsProcess,
+
                                           tGrid, xGrid, dampingSteps,
+
+
                                           schemeDesc, localVol,
+
                                           illegalLocalVolOverwrite));
        }
+
+ }
+};
+
+
+
+
%{
using QuantLib::QuantoEngine;
using QuantLib::ForwardVanillaEngine;
```