The Stochastic Dynamics of the Short Term Interest Rate in India

Jayanth Rama Varma

Reproduced with the permission of *The Indian Journal of Applied Economics*, in which the paper was first published (January-March 1997, 6(1), 47-58).

Abstract

The stochastic dynamics of interest rates is a crucial element in modern term structure theories and in the pricing of the various interest rate options which are embedded in bond issues today. International studies show that no model of these dynamics is valid world-wide.

This paper studies the dynamics of the short term interest rate in India (the call market rate) and shows that it follows a mean reverting dynamics with a volatility which is independent of the level of interest rates (conforming to the model proposed by Brennan and Schwartz, 1979). This finding has important implications for the theory of the term structure of interest rates. In particular the Cox-Ingersoll-Ross theory of the term structure is strongly rejected in India.

The normal rate of interest to which the short term rate mean-reverts is itself shown to be changing over time. The Kalman filtering methodology shows that the normal rate too follows a mean reverting process with a much slower speed of adjustment.

A companion paper (Varma, 1996b) translates these findings into a methodology for pricing interest rate options in India and presents applications of the proposed methodology.

The Stochastic Dynamics of the Short Term Interest Rate in India

Introduction

For the last few years, India has been preparing for the introduction of full fledged markets for stock and stock index options. While these markets have yet to see the light of day, options on interest rates have become increasingly important in the country's fledgling debt market. Though there is no trading in interest rate options *per se*, there has been a lot of activity in the issue of bonds with various embedded interest rate options. The pricing of the embedded call and put options, is essential to arrive at a rational valuation of these bonds.

Valuation of options on bonds is considerably more complex than the pricing of options on stocks and stock indices mainly because of the vastly greater complexity of the bond price dynamics as compared to the dynamics of stock prices. The probability distribution of stock prices closely resembles a log-normal distribution generated by a random walk. In other words, the distribution of short term stock market *returns* approximates the familiar bell-shaped normal distribution. The famous Black-Scholes option pricing formula (Black and Scholes, 1973) for valuing options on stocks is based on this distribution and is known to perform quite well in practice.

Bond prices, on the other hand, do not follow a random walk at all. In fact, as the bond approaches maturity, its price approaches the redemption value and all uncertainty rapidly disappears. (A random walk has been compared to the walking of a drunkard; the bond then is a drunkard who becomes increasingly sober as the maturity approaches!). To value options on bonds (or options embedded in them), it is, therefore, usual to regard the interest rate rather than the bond price as the underlying variable. However, the dynamics of interest rates are not straightforward either (see, for example, Malkiel, 1966). Interest rates do not follow a simple random walk, but exhibit the well known phenomenon of mean reversion. This phenomenon refers to the tendency of interest rates to revert to a normal rate over the long run. Whenever the interest rate drifts too far away from the normal rate, it is pulled back towards it. It is also well known that interest rates are more volatile when rates are high than when they are low. (To pursue our previous analogy, the drunkard's swagger increases sharply when he drifts towards the left hand side of the road and is less pronounced when he drifts towards the other end.)

A stochastic model of all this complex dynamics of the interest rate process thus becomes a pre-requisite for the valuation of interest rate options. Researchers around the world have expended a great deal of effort on this task. Unfortunately, no single model appears to be valid in all countries of the world. A recent study (Tse, 1995) of eight different models in eleven countries found that no model was valid in all countries. Each of the three most popular models found applicability in some countries, but each was rejected in half the countries. This means that we cannot simply pick up one of the models developed elsewhere in the world and apply it to India. It is necessary to study the dynamics of interest rates in India and determine the model which best fits the Indian experience.

Data and Methodology

Data

Interest rates in India have been freed only recently and therefore we do not have a long stretch of historical data to study the dynamics of interest rates. Moreover, the low level of development of Indian debt markets means that many of the bonds are highly illiquid and there is a paucity of reliable price and yield data at regular intervals. After careful examination of various alternatives, it appears quite clear that the inter-bank call market rate is the only free-market interest rate for which reliable quotations based on actual trades are available on a daily or weekly basis. The call market does have the disadvantage that it is restricted to the banks. (A limited number of other institutions can participate as lenders, but not borrowers). Nevertheless, the experience in India has been that the call market rate has a pervasive influence on all interest rates in the country. It would not be at all wrong to say that the call market is the true benchmark rate of interest in the Indian economy. This paper, therefore, proceeds to study the dynamics of the call market interest rate.

The call market rate fluctuates on a minute to minute basis and is quoted in all business newspapers everyday. Unlike in the case of stock prices, however, it makes very little sense to use the closing quotation of the day as the call market interest rate. This is because the bulk of the call market transactions take place in the first hour or so, and later quotations are typically based on very thin volumes. It would not at all be appropriate to rely on these stray late afternoon transactions. For this reason, this paper relies on the weighted average rate published by the Reserve Bank of India (RBI). Since this is arrived at by weighting the interest rate in each transaction by the actual volume of the transaction, this weighted average is more representative of the interest rate prevalent in the economy. These weighted average rates are published by the RBI on a weekly basis. This data was collected for the calendar years 1993, 1994 and 1995 from various issues of the *Reserve Bank of India Bulletin* as well as its weekly statistical supplement. The decision not to go back beyond 1/1/1993 was taken to avoid any contamination by the effect of the security scam and its immediate aftermath.

Methodology

As already pointed out, as many as eight alternative models of interest rate dynamics have been proposed in the literature. However, as pointed out by Chan(1992) and Tse(1995), all the important alternative models (in their discrete time versions) are special cases of the following equation:

$$\Delta r_t = \alpha + \beta r_{t-1} + \sigma (r_{t-1})^{\gamma} u_t \qquad \textbf{(1)}$$

where r_t is the interest rate at time *t*, $\Delta r_t = r_t - r_{t-1}$ is the change in the interest rate during time period t, α and β are parameters which specify the deterministic drift of the interest rate process, γ and σ are parameters which specify the variance of the random disturbance, u_t is a mean zero, unit variance disturbance which is assumed to be independently (normally) distributed. Eq (1) can be regarded as the discrete time approximation to the corresponding continuous time diffusion equation ($dr = a + br + \sigma r^{\gamma} dz$) when the time period t is short. In this study, the time period is, indeed, quite short - a week.

By fixing the values of one or more of the parameters in Eq (1), we can obtain (the discrete time versions of) virtually all the important models of interest rate dynamics that have been proposed in the literature:

- Merton (1973), $\beta = \gamma = 0$
- Vasicek (1977), $\gamma = 0$
- Dothan (1978), $\alpha = \beta = 0$, $\gamma = 1$
- Brennan and Schwartz (1979), $\gamma = 1$
- Cox, Ingersoll and Ross (1980), $\alpha = \beta = 0$, $\gamma = 1.5$
- Cox, Ingersoll and Ross (1985), $\gamma = 0.5$
- Geometric Brownian Motion, $\alpha = 0$, $\gamma = 1$
- Constant Elasticity of Variance, $\alpha = 0$

The mean reversion phenomenon implies that α is positive and β is negative, in which case, Eq **(1)** can be rewritten in the intuitively more meaningful form:

$$\Delta r_t = \kappa(\mu - r_{t-1}) + \sigma(r_{t-1})^{\gamma} u_t \qquad \textbf{(2)}$$

where $\kappa = -\beta$ represents the "speed of adjustment" and $\mu = -\alpha / \beta$ is the "normal rate of interest" to which interest rates revert. A κ of 1 represents full adjustment in the sense that a deviation from the normal rate of interest is fully corrected the next period while κ equal to 0 ($\alpha = \beta = 0$ in terms of Eq **(1)**) represents no mean reversion at all.

In both Eq **(1)** and Eq **(2)**, γ (which is usually required to be non negative) captures the tendency of interest rates to fluctuate more when rates are high than when they are low. The higher γ is, the greater this tendency; a γ of zero indicates the complete absence of any such tendency; in other words, the standard deviation of the changes in interest rates is independent of the level of the rates. In the finance literature, greater importance is attached to the standard deviation of the relative changes in interest rates, or, in the continuos time limit, the standard deviation of the logarithm of interest rates; this quantity is referred to as the volatility of interest rates. A γ of 1.0 indicates that the volatility of the interest rate is independent of its level. A γ above unity indicates that the volatility rises with the level of interest rates. A γ below unity indicates that the volatility falls as the level of interest rates rises.

Since all the important models proposed in the literature are obtained by fixing the values of one or more of the parameters in Eq **(1)**, these models can be regarded as hypotheses about the values of the parameters in this equation. Under the assumption that the disturbance term is normally distributed, it is possible to estimate Eq **(1)** by the method of maximum likelihood. The hypotheses implied by the various competing models can be tested by estimating the restricted models also by maximum likelihood and using the likelihood ratio test. Chan (1992) has proposed the generalized method of moments as an alternative to the maximum likelihood method to avoid distributional assumptions. But the main purpose of studying interest rate dynamics is to help in option pricing, where the distributional assumption of normality is usually made. Maximum likelihood, therefore, presents itself as the method of choice.

Results and Conclusion

The maximum likelihood estimate of Eq **(1)** is as follows:

α = 1.331, β = -0.124, γ = 0.824, σ = 0.476

implying estimates of κ = 0.124 and μ = 10.737 in Eq **(2)**.

Coming to the various restricted models, the Brennan Schwartz model could not be rejected at even the 10% level while all the other alternatives could be rejected at the 1% level or better:

- Merton (1973), $\beta = \gamma = 0$, rejected at the 0.01% level, chi-square (2 df) = 65.742, $P < 0.0001$.

- Vasicek (1977), $\gamma = 0$, rejected at the 0.01% level, chi-square (1 df) = 52.738, P < 0.0001.
- Dothan (1978), $\alpha = \beta = 0$, $\gamma = 1$, rejected at the 1% level, chi-square (3 df) = 13.742, P = 0.004.
- Brennan and Schwartz (1979), $\gamma = 1$, cannot be rejected at even the 10 % level, chi-square (1 df) = 2.26, P = 0.129.
- Cox, Ingersoll and Ross (1980), $\alpha = \beta = 0$, $\gamma = 1.5$, rejected at the 0.01% level, chi-square (3 df) = 1216, P < 0.0001.
- Cox, Ingersoll and Ross (1985), $\gamma = 0.5$, rejected at the 1% level, chi-square (1 df) = 8.072, P = 0.005.
- Geometric Brownian Motion, $\alpha = 0$, $\gamma = 1$, rejected at the 1% level, chi-square (2 df) = 11.342, P = 0.004.
- Constant Elasticity of Variance, $\alpha = 0$, rejected at the 1% level, chi-square (1 df) = 7.742, P = 0.006.

It is interesting to observe that the Cox, Ingersoll and Ross (1985) square-root process which is quite popular both as a theory of the term structure of interest rates and as an options pricing model, is strongly rejected in the Indian context in favour of the Brennan-Schwartz model. Internationally, the Tse (1995) study showed that Brennan-Schwartz was applicable in Holland, France, Australia, Belgium, Germany and Japan while being rejected in the US, Canada, Switzerland, UK and Italy.

Since the Brennan-Schwartz model has been seen to be applicable in India, all further analysis in this paper uses this model rather than the more general Eq **(1)**. The estimate for the Brennan-Schwartz model is as follows:

$\kappa = 0.126$, $\mu = 10.710$ and $\sigma = 0.326$.

The estimates for κ and μ are practically the same as in the unrestricted model while the estimate of σ adjusts downward to account for the higher value of γ. The interpretation of the parameter estimates is as follows. The estimate of μ means that the normal rate of interest is about 10.7%. Whenever the interest rate drifts above this level, it gets pulled down, and whenever it drifts below this level it gets pulled up. As stated earlier, κ is the speed of adjustment which tells us how rapidly the interest rate returns to its normal level after a disturbance. The estimated value of 0.126 for this speed of adjustment implies that only about 12.6% of a disturbance is eliminated in one week and that it would take 8 weeks for the disturbance to be completely eliminated. The parameter σ in the Brennan-Schwartz model is the volatility of interest rates. The high value for this parameter (weekly volatility of 32% corresponding to an annualized volatility of over 200%) is in line with the widespread perception that call rates in India are highly volatile.

Dynamics of the Normal Rate of Interest

It is known from international experience that even the normal interest rate does not remain unchanged for long periods of time. It too may change, but far more slowly than the actual interest rate itself. To test for this possibility, the sample period of three years was divided into three sub-periods of one calendar year each and the Brennan-Schwartz model was re-estimated with the normal interest rate μ allowed to take different values in each year. The estimated normal rates were 6.95%, 8.58% and 15.82% for 1993, 1994 and 1995 respectively. The likelihood ratio test indicates that the hypothesis that the normal rate was the same in all three years can be rejected at the 5% level (chi-square with 2 df = 7.118, P = 0.028).

Very clearly, therefore, a realistic model of interest rate dynamics in India must allow the normal rate of interest to vary. Ideally, the normal rate must be allowed to vary gradually rather than jump abruptly from one level to another every year. In other words we would like to model the interest rate dynamics as follows:

$$\Delta r_t = \kappa(\mu_t - r_{t-1}) + \sigma\, r_{t-1}\, u_t \qquad \textbf{(3)}$$

$$\Delta \mu_t = \varphi(\mu - \mu_{t-1}) + \omega\, \mu_{t-1}\, v_t \qquad \textbf{(4)}$$

Eq **(3)** differs from Eq **(2)** only in that γ is set equal to unity (as required by the Brennan-Schwartz model) and in that a time varying normal rate of interest μ_t replaces the fixed rate μ. Eq **(4)** models the evolution of the normal rate using dynamics similar to that of the actual rate itself. The normal rate μ_t itself is allowed to mean-revert to a grand normal rate μ with a speed of adjustment φ. The disturbance term v_t is assumed to be independently (normally) distributed with zero mean and unit variance. Therefore, the parameter ω is the volatility of the normal rate.

This formulation assumes that γ is equal to unity not only in Eq **(3)**, but also in Eq **(4)**. It is possible instead to let γ be a free parameter in Eq **(4)** to be estimated from the data, but there are strong reasons for not doing so. First, if γ is not equal to unity in Eq **(4)**, then the normal rate would have a level dependent volatility. From Eq **(3)** and Eq **(4)**, it is clear that a disturbance in the normal rate would transmit itself to the actual short term rate itself over a period of time. The volatility of the actual rate would therefore depend on the level of the normal rate. Since the actual rate mean-reverts to the normal rate, the levels of these two rates are correlated and the net result should be a level dependent volatility of the actual rate itself (γ not equal to unity) in Eq **(2)**. Hence having accepted the hypothesis that γ is equal to unity in Eq **(2)**, we have to make the same assumption in Eq **(4)**. Quite apart from this theoretical argument, it is doubtful whether the short sample period would be sufficient to estimate a second-moment parameter like γ with any degree of precision in Eq **(4)** since the normal rate changes far more slowly than the actual rate.

The pair of equations (Eq **(3)** and Eq **(4)**) lends itself to estimation by the method of Kalman filtering and maximum likelihood (see, for example, Chow, 1984 or Harvey, 1989). The Kalman filtering model is usually formulated as follows:

$$
\begin{aligned}
y_t &= x_t \beta_t + \varepsilon_t \\
\beta_t &= \alpha + M \beta_{t-1} + \eta_t \\
\mathbf{VAR}(\varepsilon_t) &= \sigma^2 \zeta_t^{\,2} \\
\mathbf{VAR}(\eta_t) &= \sigma^2 \xi_t^{\,2} P
\end{aligned}
\qquad \textbf{(5)}
$$

where y is the dependent variable, x a row vector of k independent variables including possibly the constant term, β a time varying column vector of k regression coefficients, and the subscript t denotes time which runs from 1 to T.

Under the assumption that ε and η are normally distributed and serially uncorrelated, Eq **(5)** can be estimated recursively as follows for fixed α, M, P and σ starting from an initial estimate of β and its variance $\sigma^2 R$:

$$
\begin{aligned}
&\text{Compute Prior Estimates:} \\
&\beta_{t|t-1} = \alpha + M \beta_{t-1|t-1} \\
&R_{t|t-1} = M R_{t-1|t-1} M' + \xi_t^2 P \\
&\text{Compute One-Step Prediction Error:} \\
&u_t = y_t - x_t \beta_{t|t-1} \\
&\text{Compute Posterior Estimates:} \\
&\beta_{t|t} = \beta_{t|t-1} + K_t u_t \\
&R_{t|t} = R_{t|t-1} - R_{t|t-1} x_t' w_t x_t R_{t|t-1} \\
&\text{where} \\
&K_t = R_{t|t-1} x_t' w_t \\
&w_t = \left[x_t R_{t|t-1} x_t' + \zeta_t^2 \right]^{-1}
\end{aligned}
\qquad \textbf{(6)}
$$

The estimation begins by computing the prior estimates denoted by the subscript $t|t\text{-}1$ which is the estimate at time t using the data only up to time t-1. We then look at the data at time t, compute the one step prediction error and revise the estimates to get the posterior estimates denoted by the subscript $t|t$ which is the estimate at time t using the data up to time t. We then move on to the next time period. The reader is referred to Chow (1984) or Harvey (1989) for the methodology for obtaining an initial estimate of $\beta_{k|k}$ and $R_{k|k}$ where k is the number of independent variables.

In this entire process, the "hyper-parameters" α, M, P and σ were kept fixed. To estimate these hyper-parameters, we resort to the method of maximum likelihood. Under the further assumption that the x vector does not include any lagged dependent variables, the one step prediction error, u, in Eq **(6)** is normally distributed and serially uncorrelated. The likelihood function can be easily derived based on this fact. The concentrated likelihood function after eliminating σ is given by:

$$\log L^* = \text{constant} - \frac{1}{2}(T-k)\log \sum_{t=k+1}^{T} w_t u_t^2 - \frac{1}{2}\sum_{t=k+1}^{T} \log w_t \qquad (7)$$

The maximum likelihood estimates of the hyper-parameters, α, M and P, are obtained by maximizing the above log likelihood with respect to these parameters. A numerical non-linear maximization procedure must be used for this purpose. The maximum likelihood estimate of the hyper-parameter σ is given by:

$$\sigma^2 = \frac{1}{T-k} \sum_{t=k+1}^{T} u_t^2 \qquad (8)$$

To apply the above Kalman filtering model to our model, the natural approach would be to set

$$y_t = r_t - r_{t-1}, \quad x_t = [1 \quad r_{t-1}], \quad \beta = [\kappa\mu_{t-1} \quad -\kappa]$$

and so forth. The difficulty with this straight forward approach is that *x* includes lagged dependent variables rendering the likelihood function (Eq **(7)**) invalid. To side-step this difficulty, we make use of the fact that the coefficient of the lagged dependent variable is constant and the time varying coefficient applies only to the constant term. We do not therefore need the Kalman filter to estimate the coefficient of the lagged dependent variable which can be treated as another hyper-parameter in the filtering process. The revised formulation is

$$y_t = r_t - r_{t-1} + \kappa r_{t-1}, \quad x_t = 1, \quad \beta_t = \kappa \mu_{t-1}, \quad M = 1 - \phi, \quad \alpha = \kappa\mu\phi$$
$$\zeta_t = r_{t-1}, \quad \xi_t = \hat{\mu}_{t-1}, \quad P = \omega^2$$

where, in the variance term, ξ_t, we have replaced the unobserved μ_{t-1} by its estimate $\hat{\mu}_{t-1}$. This formulation evidently satisfies the assumptions of the Kalman filter model; the normality and serial independence of ε and η follow from the normality and serial independence of u and v in Eq **(3)** and Eq **(4)**.

As already indicated, the parameter κ is treated as a hyper-parameter and is estimated by maximum likelihood along with the other hyper-parameters α, M, P and σ. The actual parameters of interest in our original pair of equations (Eq **(3)** and Eq **(4)**) can then be computed as follows:

$$\phi = 1 - M, \quad \mu = \alpha / \kappa\phi.$$

Applying this estimation process leads to the following estimates:

φ = 0.0392, μ = 12.12, ω = 0.190, κ = 0.718 and σ = 0.2376.

The hypothesis that φ = 0 can be rejected at the 10% level and is at the borderline of rejection at the 5% level(chi-square with 1 df = 3.534, P = 0.057). However, if φ = 0 is regarded as a boundary of the parameter space (i.e., if negative values of φ are not allowed), then the chi-square test has to be modified using the Chernoff correction (see, for example, Chow, 1984 or Harvey, 1989); in such a case, the hypothesis is comfortably rejected at the 5% level (equal mixture of chi-squares with 0 and 1 df = 3.534, P = 0.028). The hypothesis that $\varphi = 1$ can be rejected at 1% level (chi-square with 1 df = 8.014, P < 0.005). The hypothesis that $\omega = 0$ (i.e., that the normal rate is actually constant) can be rejected at the 0.1% level (chi-square with 1 df = 11.498, P < 0.001). Of course, the chi-square test needs to be corrected in this case (in fact, even the Chernoff correction is not enough), but the effect of this correction would only be to reject the hypothesis even more strongly.

Though the hypothesis of no mean reversion in the normal rate ($\varphi = 0$) is rejected only weakly (border line of rejection at the 5% level), it is certainly desirable on theoretical grounds to include the mean reversion in the model specification. In fact, it appears clear that the short sample is the principal reason for the failure to reject lack of mean reversion more strongly. Since the speed of the mean reversion is very slow ($\varphi = 0.0392$ implies that a disturbance in the normal rate would take about half a year to correct), a sample size considerably longer than 3 years would be needed to establish mean reversion more firmly.

Implications for Term Structure of Volatility

The term structure of volatility plays an important role in modern option pricing theory (Heath *et al*, 1992) and is an important input into many models for pricing interest rate options. (Just as the term structure of interest rates gives the yields for various maturities, the term structure of volatility gives the volatilities of interest rates at various maturities). Though ideally, the term structure ought to be estimated from historical data, this may not be a feasible proposition in India given the lack of reliable data going sufficiently long back in time. Estimating the term structure of volatility directly from the estimated interest rate dynamics may be an attractive alternative. The dynamics of interest rates as it emerges from this study is perhaps too complex to admit of any simple closed form solution for the term structure of volatility. The methodology of Monte Carlo simulation can however be adopted to estimate the term structure of volatility from the estimated dynamics as discussed in Varma (1996a).

Conclusion

The following important conclusions emerge from this study:

- The short term interest rate in India (the call market rate) follows a mean reverting dynamics with a volatility which is independent of the level of interest rates (conforming to the model proposed by Brennan-Schwartz).
- This finding has important implications for the theory of the term structure of interest rates. In particular the Cox-Ingersoll-Ross theory of the term structure is strongly rejected in India.
- The normal rate of interest to which the short term rate mean-reverts is itself changing over time. It too follows a mean reverting process with a much slower speed of adjustment.

The principal applications of these findings is in the field of option pricing. The dynamics of the short rate rule out some of the well known models of option pricing including Cox-Ingersoll-Ross. A companion paper (Varma, 1996b), argues that the Black-Derman-Toy model (Black *et al*. 1994) is the most attractive model for pricing interest rate options under Indian conditions.

The findings of this study are also extremely useful in estimating the term structure of volatility in India. Many popular option pricing models (including Black-Derman-Toy) require the term structure of volatility as one of the inputs to the model. Varma (1996a) shows how the estimated dynamics of interest rates reported in this paper can be used to estimate the term structure of volatility by using Monte Carlo simulation.

References

Black, F., E. Derman, W. Toy and J. C. Francis (1994) "Using a One-Factor Model to Value Interest Rate-Sensitive Securities: With an Application to Treasury Bond Options", in Francis, J. C. and A. S. Wold, ed., *The Handbook of Interest Rate Risk Management,* New York, Irwin.

Black, F. and M. Scholes (1973), "The Pricing of Options and Corporate Liabilities", *Journal of Political Economy*, 81, 637-654.

Brennan, M. J. M. and E. S. Schwartz (1979), "A Continuous Time Approach to the Pricing of Bonds", *Journal of Banking and Finance*, 3, 133-155.

Chan, K. C., G. A. Karoyli, F. A. Longstaff and A. B. Sanders (1992), "An Empirical Comparison of Alternative Models of the Short Term Interest Rate", *Journal of Finance,* 47, 1209-1227.

Chow, G. C. (1984), "Random and Changing Coefficient Models", in Griliches, Z. and M. D. Intrilligator, ed., *Handbook of Econometrics,* Amsterdam, North Holland.

Cox, J. C., J. E. Ingersoll and S. A. Ross (1985), "A Theory of the Term Structure of Interest Rates", *Econometrica*, 53, 385-407.

Cox, J. C., J. E. Ingersoll and S. A. Ross (1980), "An Analysis of Variable Rate Contracts", *Journal of Finance*, 35, 389-403.

Dothan, U. L. (1978), "On the Term Structure of Interest Rates", *Journal of Financial Economics,* 6, 59-69.

Harvey, A. C. (1989), Forecasting, structural time series models and the Kalman filter, Cambridge, Cambridge University Press.

Heath, D., R. Jarrow and A. Morton (1992), "Bond Pricing and the Term Structure of Interest Rates: A New Method for Contingent Claim Valuation", *Econometrica*, 60(1), 77-105.

Malkiel, B. G. (1966), *The Term Structure of Interest Rates; Explanations and Behaviour Patterns,* Princeton, Princeton University Press.

Merton, R. C. (1973), "A Theory of Rational Option Pricing", *Bell Journal of Economics and Management Science*, 4, 141-183.

Tse, Y. K. (1995), "Some International Evidence on the Stochastic Behaviour of Interest Rates", *Journal of International Money and Finance*, 14(5), 721-738.

Varma, J. R. (1996a), "Indian Interest Rate Dynamics and the Pricing of Options", UTI-IIMB Centre for Capital Market Education and Research, Working Paper 1/96, Indian Institute of Management, Bangalore, May 1996.

Varma, J. R. (1996b), "Bond Valuation and the Pricing of Interest Rate Options in India", *ICFAI Journal of Applied Finance,* 2(2), 161-176.

Vasicek, O. (1977), "An Equilibrium Characterization of the Term Structure", *Journal of Financial Economics,* 5, 177-188.