Mispricing of Volatility
in the Indian Index Options Market

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Mispricing of Volatility in the Indian Index Options Market

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Abstract

This paper examines the relationship between index futures and index options prices in India. By using futures prices, we eliminate the effect of short sale restrictions in the cash market that impede arbitrage between the cash and derivative markets. We estimate the implied (risk neutral) probability distribution of the underlying index using the Breeden-Litzenberger formula on the basis of estimated implied volatility smiles. The implied probability distribution is more highly peaked and has (with one exception) thinner tails than the normal distribution or the historical distribution. The market appears to be underestimating the probability of market movements in either direction, and thereby underpricing volatility severely. At the same time, we see some overpricing of deep-in-the-money calls and some inconclusive evidence of violation of put-call-parity. We also show that the observed prices are rather close to the average of the intrinsic value of the option and its Black-Scholes value (disregarding the smile). This is another indication of volatility underpricing.

1   Introduction

It is well known that severe mispricing prevails in India's nascent derivatives market. The mispricing that has been most commented upon is the negative cost of carry phenomenon in which the futures trades at a discount to the underlying. This is perhaps partly explained by the short sale restrictions in the cash market that

precludes the reverse cash and carry arbitrage that would normally eliminate this mispricing. Globally, also, it has been observed that futures trade below fair value (though not usually below underlying) in the presence of acute short sale restrictions[1].

This paper does not deal with the issue of the cost of carry at all. It sidesteps this problem by looking at the relationship between futures and options prices (given by the Black formula (Eq (1) – (4) below rather than the Black Scholes formula, Eq (5) – (8) below). In this relationship, the cost of carry is almost irrelevant. The results in this paper shows that even after removing the effect of the cost of carry in this manner, there is severe mispricing in the options market. With one exception (deep in-the-money-calls), volatility is severely underpriced. There is also some evidence that put-call parity[2] may be systematically violated in the index options market, but the evidence is not conclusive.

The plan of the paper is as follows. Using closing Nifty futures and options prices from June 2001 to February 2002, we employ the Black formula to calculate the implied volatility for each option on each day. We then fit a volatility smile to these implied volatilities separately for put and call options. Statistical tests firmly establish that the smiles are sharply different for calls and puts while put call parity requires that the smiles be the same[3]. Finally, we use the Breeden-Litzenberger formula to compute the implied probability distributions for the terminal stock index price from these two smiles. The implied probability distribution is more highly peaked and has (with one exception) thinner tails than the normal distribution or the historical distribution. The market appears to be underestimating the probability of market movements in either direction. This is the severe underpricing of volatility referred to above. The exception is in the case of deep-in-the-money-calls where a fat left tail leads to overpricing. The overpricing is not large in percentage terms because these options have large intrinsic values anyway. We also show that the observed prices are rather close to the average of the intrinsic value of the option and its Black-Scholes value (disregarding the smile).

2   Implied Volatilities

The Black formula for call and put options in terms of futures prices is as follows:

$$c = e^{-rt} \left[F\, N(d_1) - X\, N(d_2) \right] \tag{1}$$

$$p = e^{-rt} \left[X\, N(-d_2) - F\, N(-d_1) \right] \tag{2}$$

$$d_1 = \frac{\ln(F/X) + \sigma^2 t/2}{\sigma\sqrt{t}} \tag{3}$$

$$d_2 = d_1 - \sigma\sqrt{t} \tag{4}$$

where c and p are the call and put prices, r is the risk free interest rate, t is the time to expiry, F is the futures price for the same maturity, N denotes the cumulative normal distribution function and σ is the volatility of the futures price. These formulas may be compared with the Black-Scholes formulas:

$$c = S\,N(d_1) - Xe^{-rt}\,N(d_2) \tag{5}$$

$$p = Xe^{-rt}N(-d_2) - S\,N(-d_1) \tag{6}$$

$$d_1 = \frac{\ln(S/X) + rt + \sigma^2 t/2}{\sigma\sqrt{t}} \tag{7}$$

$$d_2 = d_1 - \sigma\sqrt{t} \tag{8}$$

It may be seen that the Black prices are exactly what one would obtain if we used the Black Scholes formula with the stock price S replaced by $e^{-rt}F$. This equivalence is useful if we want to get Black formula prices using a Black Scholes options calculator.

It will be noted that the risk free rate has a very small impact on the option price in the Black formula because it appears in a discounting factor that is applied after taking the difference of the two terms inside the bracket. Even setting it to zero will make a difference of less than 1% for most option prices. In the Black Scholes formula on the other hand, the discounting factor is applied to one of the terms before the subtraction, and the risk free rate makes a huge difference to option values. Put differently, in the Black-Scholes formula, the risk free rate determines whether and how much the option is in or out of money, but this does not happen in Black formula[4].

Since the results are not sensitive to the risk free rate while using the Black formula, we have used a constant risk free rate of 9% to compute option prices and implied volatilities from the Black formula[*].

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[*] The results presented below were re-estimated using risk free rates of 0% and 100%. Qualitatively, the results were quite similar.

For about 6.5% of all calls and about 7.5% of all puts, the implied volatility was undefined because the option traded below its intrinsic value. While a few such instances are to be expected because of non synchronous trading[5] while using closing prices, the large percentage of such instances is itself prima facie indicative of mispricing of options. Analysis of the reasons for this is an area for future research. For the purposes of this paper, these options were dropped from the sample and the analysis was conducted using only options for which the implied volatility could be calculated.

3   Volatility Smiles

The volatility smile is the relationship between the implied volatility and the strike price for the same maturity. There is thus a different smile for each maturity on each day. In practice however, it is common to estimate a single smile by relating the implied volatility to the "moneyness" of the option: $\ln(F/X)\big/\sqrt{t}$. (Positive values of the moneyness indicate that a call option is in-the-money and a put option is out of the money.) Note also that there is a *negative* relation between moneyness and strike price for a fixed futures price. For fixed maturity (and futures price), the moneyness is essentially the negative of the logarithm of the strike price. The fact that the futures price is also subsumed into the definition of the moneyness allows us to estimate a single smile for the entire time period under study.

We begin by examining a scatter diagram of the implied volatility against moneyness (Figures 1 and 2).
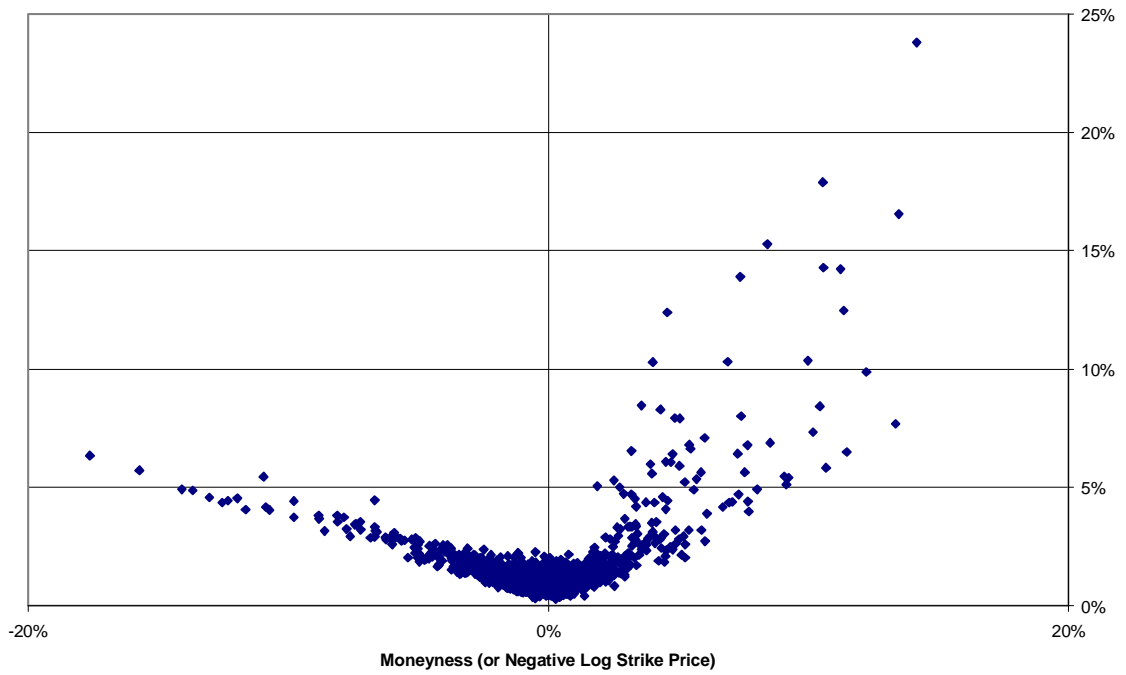
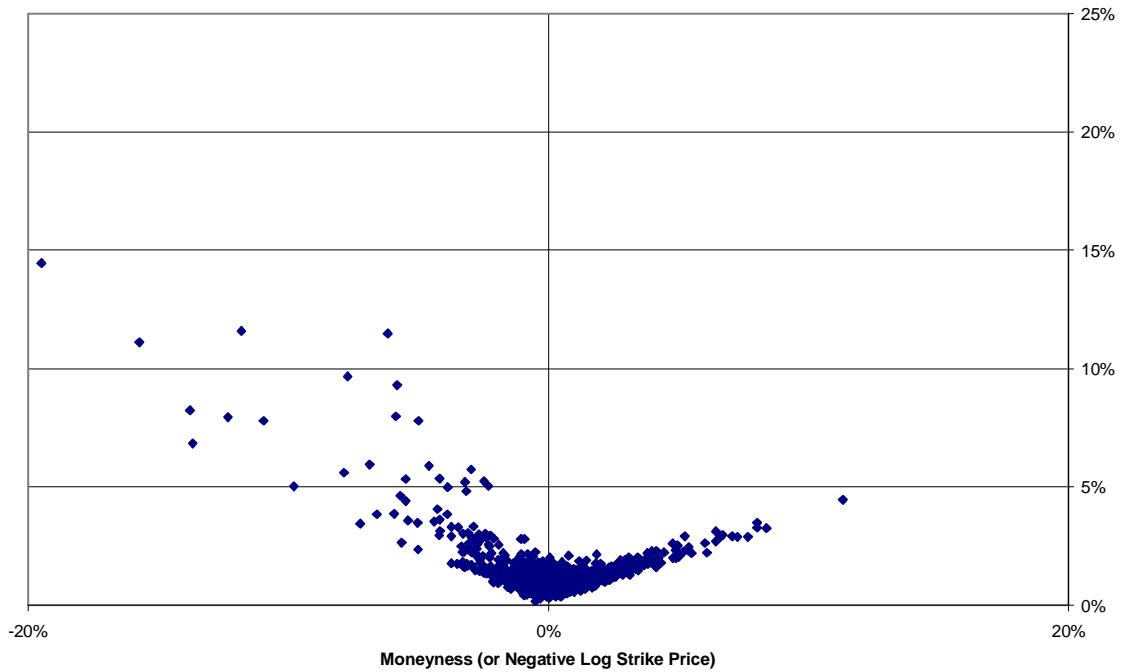**Figure 1: Call Implied Volatility**

Figure 2: Put Implied Volatility

A visual inspection reveals the following qualitative features:

- The smiles are V shaped. Normally, the smile for equity options is more like a sneer – a downward sloping curve when plotted against the strike price. Smiles in currency options are often U shaped or saucer shaped. The shapes that we observe have some similarity to that observed for currency options.

- The smiles are markedly different for puts and calls. The V is tilted towards the left for calls and tilted towards the right for puts. As already stated, put-call parity requires the two smiles to be the same. Here it is visually evident (even without the statistical tests presented later) that the smiles differ sharply.

Before proceeding to statistical analysis of the smile, it is necessary to account for time variation in the implied volatility. Not to do so could potentially vitiate the results because of the well known "omitted variable bias". To model the time variation in implied volatility we use an estimate of historical volatility obtained from the exponentially weighted moving average (EWMA) method which is widely used for this purpose. The exponential weighting coefficient for the EWMA was estimated by Nonlinear Least Squares (NLLS) to provide the best linear fit to the implied volatility. This procedure results in the following regression equation[*]:

```
V = 0.00654 + 0.51559H     R²=0.07
    (15.96)   (18.17)      F(2,4168)=330.2
```

where V is the implied volatility[†] and H is the historical volatility from an EWMA with the optimal exponential weighting coefficient of 0.83802. The regression is highly significant[‡] though the explanatory power is rather low.

The estimate from this regression is actually a GARCH estimate. To see this, we rewrite the intercept 0.00654 as 0.0134(1-0.51559), and recall that *H* is a weighted average of yesterday's volatility and today's squared return with weights 0.83802 and (1-0.83802) respectively. We see that the regression estimate is a weighted average of:

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[*] The t statistics are in parentheses

[†] The volatilities, both historical and implied are expressed on a per day basis (they have not been annualised).

[‡] Unless otherwise stated, all significance tests in this paper are at the 0.1% level (p=0.001).

- a long run volatility (1.34%) with weight (1-0.51559)

- yesterday's volatility with weight 0.51559*0.83802 and

- today's squared return with weight 0.51559* (1-0.83802).

This is therefore a GARCH model except that it is estimated by the best fit not to the actual volatility but to the implied volatility.

We now proceed to model the volatility smile by regressing the implied volatility on H, M^+ and M^- where H is as above, and M^+ and M^- are defined as max(0,M) and max(0,-M) respectively and M is the moneyness defined above as $\ln(F/X)\big/\sqrt{t}$. We estimate the regressions for call options, put options and both options together. The estimates are as follows:

	M^-	M^+	H	Intercept	R-square	F and df
Call	0.306516	0.776702	0.206408	0.004394	0.71	1968
	(27.89)	(73.32)	(8.47)	(13.01)		(4, 2421)
Put	0.602442	0.279282	0.222035	0.005246	0.78	2070
	(68.51)	(27.55)	(11.15)	(19.13)		(4, 1741)
Both	0.622162	0.43102	0.179747	0.004494	0.62	2269
	(68.78)	(49.10)	(9.14)	(16.52)		(4, 4166)

The equations have high explanatory power and are highly significant. All the coefficients and intercepts are also highly significant. Call and put options have sharply different slopes for M^- and M^+. The F-test for equality of all coefficients in the two regressions (call and put) is highly significant (F=425 with (4, 4166) df). The t-test shows that the differences in slopes for M^- and M^+ between the two regressions are statistically highly significant (t statistics of -21.02 and 33.93 respectively). These tests firmly establish the difference between the smiles for call and put options. This could also be regarded as evidence of violation of put-call parity, but we must be careful in drawing such an inference[6].

While the estimated smiles have high explanatory power and are highly significant, some further refinements appear desirable. First, the V-shape that we see in the scatter diagram has a rounded vertex while the linear regression produces a V-shape with a sharp corner. As a result, the smile is non-differentiable when the option is at the money. In the subsequent analysis of the implied probability density, we need to compute the second derivative of the option price with respect to the strike price, and the non-differentiability of the smile is a problem.

We resort to a hyperbola as a simple way to produce a rounded vertex by adding only one extra parameter. The 'V' from the linear regression is a pair of lines $y=-ax$ and $y=bx$ that can be represented by the single equation (a degenerate hyperbola) $(y+ax)(y-bx)=0$ where y denotes the implied volatility and x denotes the moneyness, M. If we add a constant c^2 to the equation, $(y+ax)(y-bx)=c^2$, we get a hyperbola with a rounded vertex that becomes increasingly flatter as c increases. The idea is that the parameter c of the hyperbola can be estimated by non-linear least squares. Solving the quadratic equation of the hyperbola for y gives the expression

$$y = \frac{-(a-b)x \pm \sqrt{(a+b)^2 x^2 + 4c^2}}{2}$$ to be estimated by NLLS. To this must be added

the expression involving historical volatility $d + \alpha H$.

On further visual inspection of the scatter diagram, we observe a faint trace of non-linearity in the arms of the V-shape. This is most evident in the right arm of the V-shape for call options. This suggests adding some quadratic terms to the equation – perhaps, the squares of M^- and M^+. Having just used a hyperbola to get rid of M^- and M^+, we do not want to give up differentiability again by putting them back in. Instead, we add ey^2 to the equation where e is another parameter to be estimated and y is the solution of the quadratic equation for the hyperbola. This technique also economises on parameters since introducing M^{-2} and M^{+2} into the equation would have added two parameters rather than the one that we have brought in.

Our equation for the implied volatility is therefore:

$$V = d + \alpha H + y + ey^2$$

$$y = \frac{-(a-b)M \pm \sqrt{(a+b)^2 M^2 + 4c}}{2}$$

involving six parameters – a, b, c, d, e and α to be estimated simultaneously by NLLS[*].

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[*] It may be noted that when M is set equal to zero, $y = c$ and the implied volatility (viewed as a regression against H) has an "intercept" $c + d + ec^2$. Thus while the estimated implied volatility subsumes a Garch model, the Garch parameters are not now easy to identify.

The linear regression estimated earlier is nested within this model ($c=e=0$) and the hyperbola without a quadratic correction ($e=0$) is nested between the two. We can therefore use a F test (or equivalently a t test since there is only additional parameter involved) to choose between these nested models. We thus get t tests for the hypotheses $c=0$ and $e=0$.

The results are as follows:

```
                   Call       Put       Both
α                0.25615   0.23982    0.25453
a                0.18652   0.64501    0.24592
b                0.41800   0.35948    0.38430
c                0.00144   0.00496    0.00282
d                0.00474   0.00162    0.00327
e               26.84434   0.00000   26.84582
R-square         0.75397   0.78985    0.67824
t-stat for e = 0   14.76      0.00      22.06
t-stat for c = 0   14.27      8.55      15.34
```

It may be seen that the hyperbolic term c is highly significant in all cases and the quadratic term e is highly significant for call options as well as for all options taken together but not for put options. While one might have expected e to be negative, so that the smile is moderated at high levels, the estimated e is positive so that the smile is actually strengthened at high levels.

The estimated smiles are plotted in Figure 3.

Figure 3: Estimated Smiles
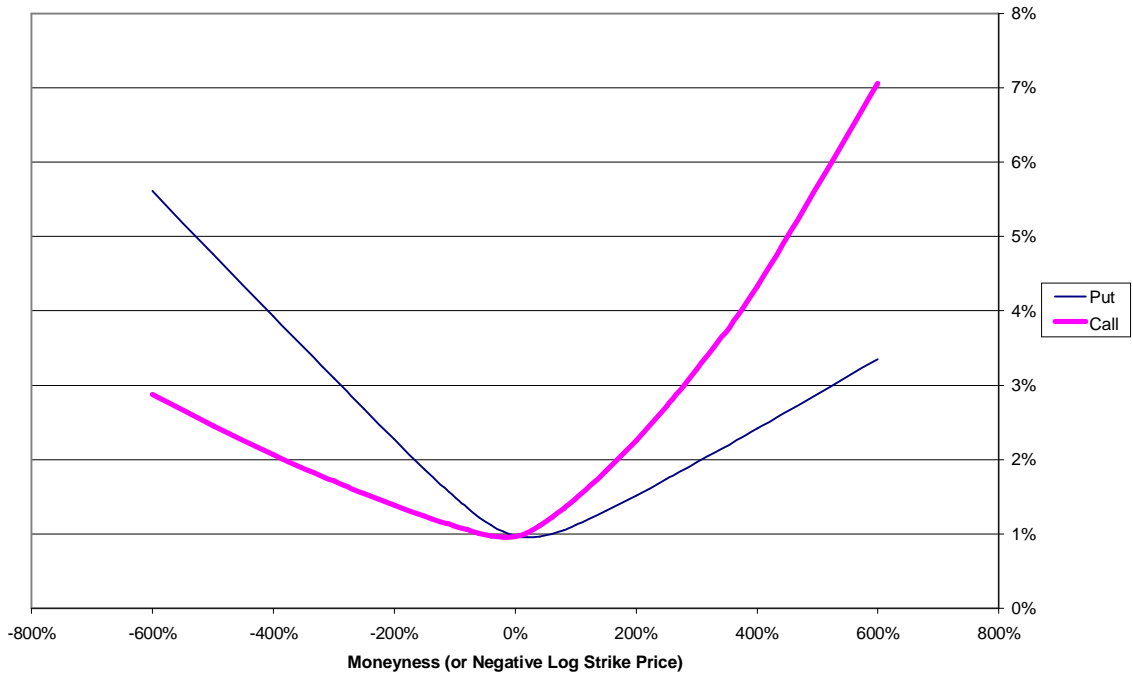
4   Goodness of Fit to Option Prices

While fitting a curve to implied volatilities is regarded as numerically superior to fitting a curve to option prices (Jackwerth, 1999), we are ultimately interested in the goodness of fit to option prices. We therefore use the fitted smiles to calculate option prices from the Black formula and compare the results with the market prices. We compare our results with three other pricing models:

- a *no smile* or flat smile model in which the same implied volatility is used for all strikes on the same day

- a *constant volatility* model in which a single long run implied volatility is used for the entire sample

- the *intrinsic* model that assigns the value max(0, *F-X*) to the call option and the value max(0, *X-F*) to the put option.

The goodness of fit is measured by regressing the actual prices on the model prices for each model separately. A good fit would be reflected in a zero intercept, unit slope and high R^2. The regression results are as follows:

	Fitted Smile	No Smile	Constant Volatility	Intrinsic Value
Intercept	0.26813	-2.57154	29.43855	10.41490
(t stat)	(2.57)	(-13.46)	(39.66)	(65.96)
Slope	0.99553	0.99479	-0.05063	0.975287
(t stat)	(412.72)	(231.85)	(-3.45)	(232.37)
R^2	0.976	0.928	0.003	0.928
F Statistic df=(1, 4168)	170338	53756	11.93	53995
t-statistic for slope = 1	1.85	1.21	71.68	5.89

The model based on the fitted smile does extremely well with an R^2 of 0.98. The intercept is not significant at the 0.1% level used throughout this paper (it just misses being significant at the 1% level). The slope is not significantly different from unity. The flat smile model has a lower R^2 than the fitted model. Moreover, it has an intercept that is highly significant. The constant volatility model fares disastrously. Most surprisingly, the intrinsic value model does as well as the no smile model.

As a further test to establish the importance of the smile, we estimate the following regression equation:

$$P_{actual} = 0.07488 + 0.04524 P_{nosmile} + 0.95259 P_{smile} \quad R^2=0.976$$
$$\quad (0.66) \quad\quad (4.26) \quad\quad\quad (91.88) \quad\quad F(2,4167)=85528$$

where as the names suggest, P_{actual} is the actual price while P_{smile} and $P_{nosmile}$ are the prices from the fitted smile model and the no smile model respectively. It is seen that the coefficient of $P_{nosmile}$, though statistically significant is very small (less than 0.05) while the coefficient of P_{smile} is more than 20 times larger and close to unity. As compared to the regression on P_{smile} alone shown in the earlier table, the R^2 remains the same up to the third decimal place (it increases by 0.0001).

Another interesting result that emerges is that both the actual prices and the fitted smile prices are quite well approximated by an equally weighted average of the intrinsic value and the no smile prices[7]. It is as if half the investors value the option at intrinsic value and the other half use the simple Black formula (without a smile) and the market price that results is the average of the two. The regression results are as follows:

$$P_{actual} = 2.95425 + 0.50875 P_{nosmile} + 0.51701 P_{intrinsic} \quad R^2=0.967$$
$$\quad (19.36) \quad\quad (69.33) \quad\quad\quad (69.07) \quad\quad F(2,4167)=60274$$

$$\mathbf{P_{smile}} = \mathbf{2.78941} + \mathbf{0.51260P_{nosmile}} + \mathbf{0.51543P_{intrinsic}} \quad \mathbf{R^2 = 0.986}$$
$$\mathbf{(28.27) \quad (108.07) \quad (106.52) \quad F(2,4167) = 60274}$$

Though the fitted smile model is so well approximated by combining the no smile and intrinsic models, it is still true that it provides a better fit to the actual prices than the combination. We regress actual prices on the three models – fitted smiles, no smile and intrinsic:

$$\mathbf{P_{actual}} = \mathbf{0.60910} + \mathbf{0.08368P_{nosmile}} + \mathbf{0.07780P_{intrinsic}} + \mathbf{0.84073P_{smile}}$$
$$\mathbf{(4.35) \quad (6.90) \quad (6.48) \quad (41.79)}$$
$$\mathbf{R^2 = 0.976. \quad F(3,4166) = 57593}$$

The coefficient of the fitted smile model is not much below unity and is about ten times the coefficients of the other two models. The R^2 is still unchanged up to the third decimal place from a regression on the fitted smile price alone (it increases by 0.0003).

The goodness of fit can also be measured in terms of the percentage absolute pricing error:

$$\frac{|actual\ price - model\ price|}{actual\ price} \times 100$$

However, in doing so it is necessary to ignore low price options as even small pricing errors result in large percentage errors for such options. We therefore ignore options whose price is less than 1% of the futures price and compute the mean and median of the percentage absolute pricing error for the remaining options for all models. For comparison we also present the mean percent error obtained when the model price is replaced by the sample mean of actual prices:

Model	Percentage absolute pricing error	
	Mean	Median
Fitted smile	14.83%	10.27%
No smile	26.04%	11.96%
Constant Volatility	76.90%	54.13%
Intrinsic value + 10[*]	25.84%	19.90%
Sample Mean Price	50.81%	43.80%

Again, the fitted model is far superior to the other models.

5   Implied Probability Density

Breeden and Litzenberger (1978) showed that a primitive security that has a unit payoff when the asset price at time *T* lies between *s* and *s+ds* is given by

$\left(\dfrac{\partial^2 c(X,T)}{\partial X^2} \right)_{X=s}$ where *c(X,T)* is the price of a call option with strike price *X* maturing

at time *T*. Multiplying this by e^{rT} gives us the risk neutral probability that the

terminal asset price lies between *s* and *s+ds*. In other words, $e^{rT}\left(\dfrac{\partial^2 c(X,T)}{\partial X^2} \right)_{X=s}$ gives

us the risk neutral probability density of the terminal stock price. By put-call parity, the Breeden-Litzenberger formula holds for put prices as well[8].

We now derive the risk neutral probabilities implied by the smiles estimated above. For various values of *X*, we compute the implied volatility using the smile, plug this into the Black-Scholes formula to get the option price. We use numerical differentiation to compute the second derivative of the option price. Since the stock price is approximately lognormally distributed, we transform the computed density to the implied density of the log stock price standardised to a zero mean and unit variance. This is the same as computing the distribution of the logarithmic stock price

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[*] The pure intrinsic model fares very badly as it assigns zero value to all out of the money options. The reasonable R^2 of this model with actual prices is because the regression equation includes an intercept of about 10.41. To give a reasonable chance to the intrinsic model, we simply add a constant 10 to the intrinsic model values. A better solution might be to consider max(10, intrinsic_value).

return. Figure 4 plots the computed implied densities from call and put options. For comparison, it also plots the standard normal density as well as the historical probability density of the Nifty index computed in Varma(1999) using data for 1990-1998.

Figure 4: Implied Probability Density of the Log Stock Return
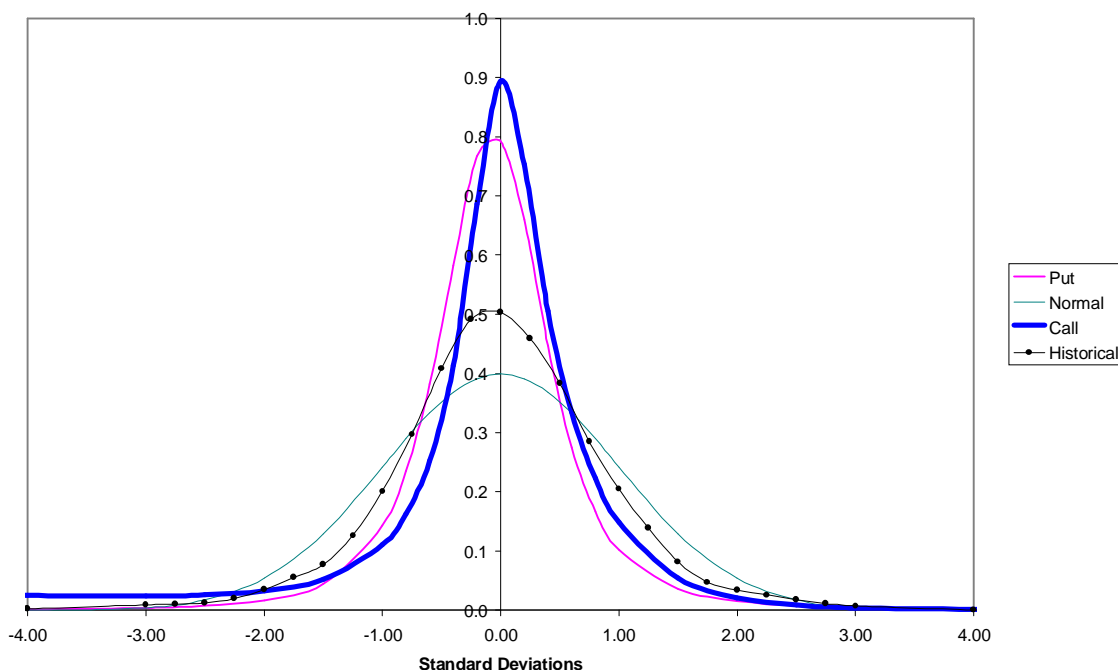
It is seen that the implied densities are much more highly peaked and (with one exception discussed later below) have thinner tails than the normal density and the historical density. The high peak and thin tails imply an expectation that the stock price would move within a very narrow range. This is inconsistent with historical experience and any plausible forecast of the future.

The only exception is the fat left tail for call options. This bears some superficial similarity to the findings of Jackwerth and Rubinstein (1996) who observed fat left tails (10 and 100 times fatter than the normal at three and four standard deviations respectively) for index options in the United States after the crash of 1987. This phenomenon of "crashophobia" as Rubinstein called it, made a big difference to the valuation of out-of-the-money put options which should be worth very little under the Black-Scholes model. However, the fat left tail that we see here does not apply to put options. The fat left tail is only for call options, it affects only deep in the money calls, and even here the percentage effect on the price is not very large as these options anyway have large intrinsic values. The fat tail here does not seem to have anything to do with "crashophobia" since we do not see a fail tail for put options.

A more intelligible explanation of the peak and tails of the implied probability densities is provided by the earlier finding that the market prices are approximately equal to an average of the Black-Scholes prices (without any smile) and the intrinsic value of the option[9]. Since the intrinsic value is based on the assumption of zero volatility, it is clear that the implied distribution would have a large peak at zero (no change in the index) and would have thin tails.

As for the deep-in-the-money calls, a plausible explanation would be that the market is starting with the intrinsic value of the option and adding a small time value to this. We saw earlier that adding a constant Rs 10 to the intrinsic value gives a tolerably good fit to actual prices. A better fit was got when we added a time value equal to half of what it should be under Black-Scholes[*]. What appears to be happening is that for deep-in-the-money calls where the Black-Scholes time value is negligible, the market still adds a small time value and those overprices them. This turns up in the analysis as the fat left tail for the call option implied distribution. Why we do not see a corresponding fat right tail for put options is a mystery.

6   Market Maturity and Learning

One might speculate that as the market becomes more mature and participants learn more about various derivative products and their inter-relationships, price discovery would improve and the observed mispricing would diminish. Some attempts were made to divide the sample into subperiods and look for improvements in pricing relationships. These attempts were not very successful.

There is evidence for a highly significant structural break after September 11, 2001. The terrorist attacks on the World Trade Centre in the United States on that day led to a sharp spike both in historical (Garch) volatilities and in implied volatilities. An intercept dummy for the post 9/11 period in the regressions for implied volatility has a positive and highly significant coefficient. The natural interpretation of this result would be that there was an upward shift in the long run volatility in the Garch model after the 9/11 episode. One could with some ingenuity argue that what happened instead was a reduction in the underpricing of volatility due to increased maturity and learning. It is however difficult to see how such learning would happen overnight. One would expect learning to more gradual and to take the form of a positive

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[*] Averaging the intrinsic and the Black-Scholes prices is equivalent to adding half the time value to the intrinsic value.

coefficient when a time variable that is added to the implied volatility regression. However, in the presence of the 9/11 dummy, the time variable is not significant at all.

All told, we would like to leave the learning hypothesis as a matter for future research. For this reason, the results relating to the 9/11 dummy and the time variable are not being reported here.

7   Conclusion

We have established severe mispricing in the Indian index options market even after using futures prices to eliminate the effect of short sale restrictions in the cash market. In particular, volatility is severely underpriced.

We have estimated volatility smiles separately for put and call options and established by statistical significance tests that the smiles are sharply different for calls and puts while put call parity requires that the smiles be the same. The implied probability distribution is more highly peaked and has (except for deep-in-the-money calls) thinner tails than the normal distribution or the historical distribution. The market thus appears to be underestimating the probability of market movements in either direction. At the same time, we see some overpricing of deep-in-the-money calls and some inconclusive evidence of violation of put-call-parity. We also show that the observed prices are rather close to the average of the intrinsic value of the option and its Black-Scholes value (disregarding the smile).

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[1] It is sometimes argued that index arbitrage is not affected by short sale restrictions as a significant amount of index arbitrage can and is done by institutions who own the stock. Neal (1996) is often cited in support of this proposition. In the body of his paper, however, Neal is more circumspect: "For the period analysed in this study, both the presence of arbitrageurs who are long in stocks and the possible circumvention of short sale restrictions suggest that the downtick rule had little effect on the mispricing. This result may not however extend to other periods. If short sale restrictions are binding, the restrictions will affect the mispricing unless institutions possess sufficient capital to fully exploit negative mispricing arbitrage opportunities". Jiang, Fung and Cheng (2001) review studies covering Finland, Germany, United Kingdom and Hong Kong that show that short sale restrictions do have an impact on the market. They also show that the lifting of short sale restrictions in Hong Kong after 1994 enhanced the dynamic efficiency of the relationship between the cash and futures markets.

[2] It may be noted that put-call parity does not require the ability to short the cash. The payoff on expiry of a call option with exercise price X and maturity T is $\max(0, S_T - X)$ where S_T is the terminal stock price. The payoff of a put option is $\max(0, X - S_T)$. We have the relationship

$$\max(0, S_T - X) = (S_T - X) + \max(0 - (S_T - X), X - S_T - (S_T - X))$$

$$= (S_T - X) + \max(X - S_T, 0)$$

$$= (S_T - X) + \max(0, X - S_T).$$

This establishes the put-call parity:

$$c(X,T) = f(X,T) + p(X,T)$$

where $c(X,T)$ is a call option with exercise price X and maturity T

$p(X,T)$ is a put option with exercise price X and maturity T

$f(X,T)$ is a security that has payoff $(S_T - X)$ at maturity T

The security $f(X,T)$ is usually thought of as a long position is the stock financed partly by borrowing an amount $X e^{-rT}$ whose repayment (principal plus interest) at time T equals X. However, the same payoff can be achieved through a position in futures instead of the underlying. Therefore, where the put-call arbitrage requires shorting $f(X,T)$, it is sufficient to be able to sell futures. Short sale restrictions on the cash market are quite irrelevant.

[3] Let $p_{BS}(X,T,\sigma)$ and $c_{BS}(X,T,\sigma)$ represent the Black-Scholes put and call prices based on a volatility σ. These prices obey put-call parity:

$$c_{BS}(X,T,\sigma) = f(X,T) + p_{BS}(X,T,\sigma)$$

As shown in above note 2 above, the put-call parity for actual market prices is:

$$c(X,T) = f(X,T) + p(X,T)$$

Let σ_X be the implied volatility at which the Black Scholes formula yields the market price of the call. Putting $\sigma = \sigma_X$ makes the left hand sides of the above two equations equal. Therefore the right hand sides of the above equations must also be equal. This means that $p_{BS}(X,T,\sigma_X) = p(X,T)$ so that σ_X is also the implied volatility of the put. Since this is true for each strike price X, the smiles computed from put and call prices must be the same whenever put-call parity holds for these prices.

[4] Often, an option is said to be in or out of the money according as the strike is below or above the current stock price, and if the option is in the money, the difference between the stock price and the strike is often said to be the intrinsic value of the money. This is not quite correct. If we let the volatility go to zero in Eq (1) - (8) above, we get the intrinsic value of a call as $e^{-rT} \max(0, F{-}X)$ or $\max(0, S - e^{-rT} X)$. Thus the moneyness of the option and its intrinsic value are related to $F{-}X$ and not $S{-}X$. (in the case of a put option, the intrinsic value is related to $X{-}F$. This is the viewpoint adopted in this paper.

[5] Trading in options was rather thin throughout this period particularly when we recognise that the modest daily trading volume is distributed over put and call options of several different strikes and maturities. This means that the closing price of many options came from trades done several hours before close. Futures prices might also be stale, but perhaps less so. Though futures trading volumes were also quite modest, futures were traded more heavily than options, and the trading volume was distributed over only three maturities. In many cases, therefore, the closing option price and futures price were not prices that prevailed simultaneously. If there are substantial intra-day price movements, an option that was above intrinsic value when it was traded early in the day might appear to be below intrinsic value when the futures closed at a different price later in the day.

[6] To establish violation of put-call parity, it is not sufficient to establish that a fitted model violates put-call parity. It is necessary to show the violation in actual prices. The difference between the two smiles does prove that any valuation model that uses the Black-Scholes model with a volatility smile would violate put-call parity. But it is possible that there are other non Black-Scholes models that provide reasonably good fits to actual prices. We present such a model later in this paper and discuss its implications for put-call parity in note 7. One may be tempted to test for put-call parity violation in the actual prices directly by looking at the prices of put and call options with the same strike price. This testing is problematic because of the phenomenon of asynchronous trading described in note 3. Put call parity is even more severely affected by asynchronous trading as the parity holds only when the prices of put, call and future are all as at the same instant of time. Any slight non simultaneity in the prices of puts, calls and futures would cause a violation of the put-call parity. The mean absolute violation of put-call parity (in the sample of over 1600 observations where puts and calls of the same strike price and maturity were traded on the same day) is about Rs 2.5 representing nearly 10% of the mean put

price. A put-call parity violation of at least Rs 5 is found in about 14% of the cases. These violations are indeed quite large but it is difficult to determine how much of these are apparent violations due to asynchronous trading and how much is real. A definitive answer on put call parity would perhaps require intra-day prices.

[7] This model has implications for put-call parity issue discussed in note 6 above. The point is that both the no smile model and the intrinsic value model separately satisfy put-call parity. (In fact, the intrinsic value model is a Black-Scholes model with zero volatility.) As such an average of the two would also satisfy the parity condition. Of course, it is true that this average still has a lower R^2 with actual prices than the fitted smile model. But the point is that it might be possible to find another model with a better fit.

[8] Put call parity states that $c = p + f$ where c and p are the call and put prices respectively while f is the price of a position involving the stock or the futures along with some borrowing. This implies that

$$\frac{\partial^2 c}{\partial X^2} = \frac{\partial^2 p}{\partial X^2} + \frac{\partial^2 f}{\partial X^2} = \frac{\partial^2 p}{\partial X^2}$$ since the stock price or futures price is independent of the exercise price.

[9] This is a bit facetious. In reality, it was an examination of the implied probability distribution that led the author to consider the idea of using intrinsic value in conjuction with a Black-Scholes model to approximate option prices.